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Research Article

Methods for multi-segment continuous cable analysis

Abdullah Demir [a,*], Uğur Polat [b]

[a] Department of Civil Engineering, Abdullah Gül University, 38080 Kayseri, Türkiye
[b] Department of Civil Engineering, Middle East Technical University, 06800 Ankara, Türkiye

ABSTRACT

Cables are invaluable members for some applications of engineering. The specialty is due to its behavior under transverse loads. Having almost no rigidity in transverse direction makes cables different from other structural elements. In most applications, cables are assumed to be two force members. However, not only its weight but also its application with roller supports makes them different structural elements. Generally, cables are assembled as single-segmented cables (SSC) where they are fixed at their ends. However, in most of the SSC applications, cables have intermediate supports which can be rollers or sliders. These type of cable applications are called as multi-segment continuous cables (MSCC). In MSCC systems, the cable fixed at its ends and supported by a number of intermediate rollers. Total length of cable is constant, and the intermediate supports are assumed to be frictionless and stationary. In this problem, the critical issue is to find the distribution of the cable length among the segments in the final equilibrium state, so reactions at all supports can be found. Two methods are proposed for the segment length adjustment based on the stress continuity among the cable. These methods are named as direct stiffness method and tension distribution method (relaxation method). Results calculated from the proposed methods are verified by both the reference benchmark problems and commercial finite element program.

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1. Introduction

Cables are invaluable elements for structural systems such as guyed towers, cable-stayed bridges marine vehicles, offshore structures, cable roofs, tensegrities, transmission lines and pre-stressing works. Cables can be bended without any residual stress. This property makes them more nonlinear than other structural elements. Although, the nonlinearity is both geometric and material, material nonlinearity is not considered (Judge et al. 2012; Prawoto and Mazlan 2012) in the scope of this research.

Various single-segment cable (SSC) analysis methods have been proposed by researchers. These methods solve the continuous cable fixed at both ends. Some researchers (Dischinger 1949; Ernst 1965) made some shape predictions for cable which is generalized in the research of Hajdin et al. (1998) and some made finite element calculations with iterative procedures which is pioneered by Micholas and Brinstiel (1962) and Skop and O'Hara (1970). After increase of computational capabilities in 1980's, researchers have proposed methods for more accurate results. Peyrot and Goulois (1979) proposed a finite element solution procedure for cable considering its catenary action. Polat (1981) applied Newton-Raphson method to the nonlinearity of cable problems. Fleming (1979) and Ren et al. (2008) proposed different finite element procedures to solve cable structures. Force density method was also used by Christou et al. (2014) for implementation of slack cables. Besides, author used (Dinçer and Demir 2020) Smoothed particle hydrodynamics (SPH), which is a meshless method, for analysis of single segment cable.

Although there are many studies about SSC analysis, limited researches have made for multi-segment continuous cables (MSCC). Some solution methods for cable systems having more than one segment were proposed

* *Corresponding author.* Tel.: +90-312-210-2401 ; Fax: +90-312-210-5401 ; E-mail address: abdullah.demir@agu.edu.tr (A. Demir)
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in the following studies. Aufaure (1993, 2000) defined a cable system having two segments. In that study, a cable was fixed at both ends and supported by one roller support. The cable was analyzed with a finite element method in which a specific element was defined. This finite element was the contact element of cable with roller support. Three nodes named as N1, N2, N3 were defined on this element in which N3 is the intermediate one. Position of N3 must be in between N1 and N2 and it is found by stress continuity through the cable. A similar method with sliding cable elements was proposed by Zhou et al. (2004). Ju and Choo (2005) are proposed a super element approach. Although frictional effect between cables and pulleys was taken into consideration, cable was assumed to be a linear structural element in that study. McDonald and Peyrot (1988, 1990) studied on cables suspended in sheaves. They used a cable element based on a catenary relationship and defined a pulley element in their study. Besides, a dynamic relaxation formulation was given for tensegrity structures by Bel Hadj Ali et al. (2017). Element free Galerkin method was also used for solution of membranes strengthen by sliding cable in research of Noguchi (2004) and Dehghan and Abbaszadeh (2016).

In this study, a novel method is proposed for the solution of multi-segment continuous cable analysis. Solution of continuous cable is achieved by dividing the complete cable system into segments. Each single-segment cable is solved by the method proposed by Polat (1981). This method is redefined for the sake of completeness of the research. Then, the direct stiffness and tension distribution method is defined (Demir 2011). Methods are verified by benchmark problems and commercial finite element solver (ANSYS).

2. Methodology

2.1. Single segment cable (SSC) analysis

Cable fixed at one end is a determinant system and the second end of cable has a position for the corresponding reaction at the first end. Reaction at the first end is changed by some iteration techniques until the released end of cable is positioned at desired location, which is the second fixed support. A detailed formulation of SSC analysis for 2D and 3D can be seen in the studies of Polat (1981) and Demir (2011), respectively.

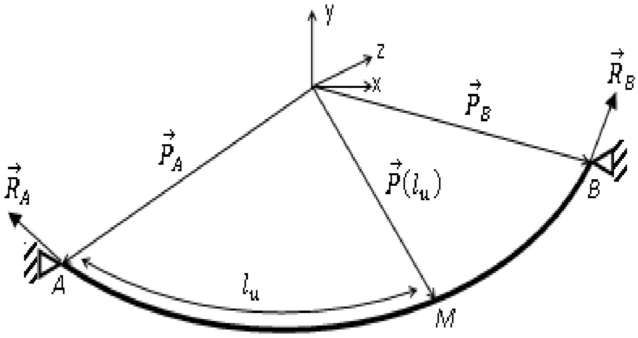

Fig. 1. An SSC layout.

In an SSC as illustrated in Fig. 1, position of an arbitrary node M can be defined as;

$$\vec{P}(l_u) = \vec{P}_A - \int_0^{l_u} \frac{\vec{R}(s)}{T(s)}[1+\varepsilon(s)]ds \tag{1}$$

where $\vec{P}_A$ is the position vector of A, l_u is the cable length from A to M, $\vec{R}(s)$ $T(s)$ and $\varepsilon(s)$ are the reaction vector, tension and strain at s, respectively.

The relation between change in position of node B, $\Delta\vec{P}_B$, and change in reaction at node A, $\Delta\vec{R}_A$, can be expressed with the help of tangent stiffness matrix $[S]$.

$$\Delta\vec{R}_A = [S]\Delta\vec{P}_B \tag{2}$$

From Equation (1) $\Delta\vec{P}_B$ is determined as;

$$\Delta\vec{P}_B = -\int_0^{L_u}\left\{\left|\frac{1+\varepsilon(l_u)}{T(l_u)}\right|\Delta\vec{R}_A - \left|\frac{1+(1-\upsilon)\varepsilon(l_u)}{T^3(l_u)}\right|\left[\vec{R}(l_u)\cdot\Delta\vec{R}_A\right]\vec{R}(l_u)\right\}dl_u \tag{3}$$

where υ is Poissson's ratio.

In global coordinate directions, Eq. (3) can be expressed in Cartesian coordinate directions as;

$$\Delta P_{BX}\hat{\imath} = -\int_0^{L_u}[C_1\Delta R_{AX}\hat{\imath} - C_2C_3C_4]\,dl_u \tag{4a}$$

$$\Delta P_{BY}\hat{\jmath} = -\int_0^{L_u}[C_1\Delta R_{AY}\hat{\jmath} - C_2C_3C_4]\,dl_u \tag{4b}$$

$$\Delta P_{BZ}\hat{k} = -\int_0^{L_u}[C_1\Delta R_{AZ}\hat{k} - C_2C_3C_4]\,dl_u \tag{4c}$$

where

$C_1 = \left[\frac{1+\varepsilon(l_u)}{T(l_u)}\right]$,

$C_2 = \left[\frac{1+(1-\upsilon)\varepsilon(l_u)}{T^3(l_u)}\right]$,

$C_3 = [R_X(l_u)\Delta R_{AX} + R_Y(l_u)\Delta R_{AY} + R_Z(l_u)\Delta R_{AZ}]$, and

$C_4 = [R_X(l_u)\hat{\imath} + R_Y(l_u)\hat{\jmath} + R_Z(l_u)\hat{k}]$.

where ΔP_{BX}, ΔP_{BY} and ΔP_{BZ} are directional components of $\Delta\vec{P}_B$ as for $\Delta\vec{R}_A$, L_u is total unstressed length of cable, $R_X(l_u)$, $R_Y(l_u)$ and $R_Z(l_u)$ are directional components of $\vec{R}(l_u)$.

Writing Eqs. (4a), (4b) and (4c) in the form of Eq. (2);

$$\begin{Bmatrix}\Delta P_{BX}\\ \Delta P_{BY}\\ \Delta P_{BZ}\end{Bmatrix} = [S]^{-1}\begin{Bmatrix}\Delta R_{AX}\\ \Delta R_{AY}\\ \Delta R_{AZ}\end{Bmatrix} \tag{5}$$

where

$$[S] = \begin{bmatrix} -\int_0^{L_u}[C_1 - C_2R_X^2(l_u)]\,dl_u & -\int_0^{L_u}[C_2R_X(l_u)R_Y(l_u)]\,dl_u & -\int_0^{L_u}[C_2R_X(l_u)R_Z(l_u)]\,dl_u \\ -\int_0^{L_u}[C_2R_Y(l_u)R_X(l_u)]\,dl_u & -\int_0^{L_u}[C_1 - C_2R_Y^2(l_u)]\,dl_u & -\int_0^{L_u}[C_2R_Y(l_u)R_Z(l_u)]\,dl_u \\ -\int_0^{L_u}[C_2R_Z(l_u)R_X(l_u)]\,dl_u & -\int_0^{L_u}[C_2R_Z(l_u)R_Y(l_u)]\,dl_u & -\int_0^{L_u}[C_1 - C_2R_Z^2(l_u)]\,dl_u \end{bmatrix} \tag{6}$$

An iterative solution of Eq. (5) gives the solution for SSC. Results of SSC are important because MSCC analysis is based on it, which means that obtaining correct SSC results will calibrate the MSCC analysis. This relation is more apparent in MSCC part. Two reference case (in part 3) are used to validate the results of SSC.

2.2. Direct stiffness method (DSM)

Multi-segment continuous cables are monolithic structural elements like continuous beams. In MSCC system there are number of intermediate supports. These supports are stationary and frictionless. Thus, cable is free to slide over these intermediate supports. In addition to the assumption of zero friction, intermediate rollers are assumed to be points. Thus, cable finds its stationary position by sliding on the roller supports i.e. changing length of cable at each segment. Direct stiffness method is developed by modeling this inherent sliding motion.

Total cable length of a MSCC system is known. However, length of each segment is unknown. Therefore, solution procedure of MSCC system starts with distribution of total cable length to each segment. In Fig. 2, an initial geometry of MSCC is given with defined unstressed segment lengths l_u^i, where i is the segment number.

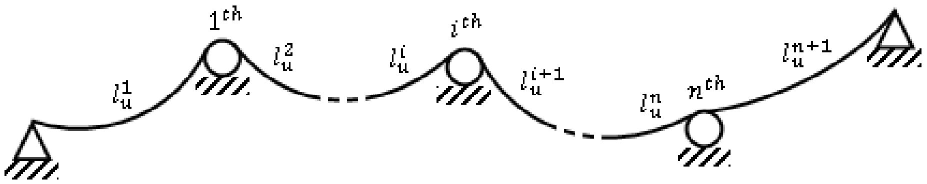

Fig. 2. Configuration of an MSCC system.

Summation of each unstressed cable length gives the total length of the continuous cable with n segments.

$$L_u = \sum_{i=1}^{n} l_u^i \tag{7}$$

Solution of each cable segment is performed by SSC procedure with its known cable length. SSC solution for each segment gives the forces at the ends of the segments. Wrong distribution of segmental lengths will lead to unbalanced forces on intermediate roller supports. The unbalanced forces at i^{th} roller support (i^{th} roller support is the connection point of i^{th} segment and $(i+1)^{th}$ segment) is shown in Eq. (8) as ΔT^i.

$$\Delta T^i = \left|\vec{R}_F^{i+1}\right| - \left|\vec{R}_L^{i}\right| \tag{8}$$

where $\vec{R}_F^{i+1}$ and $\vec{R}_L^{i}$ are shown in **Hata! Başvuru kaynağı bulunamadı.**.

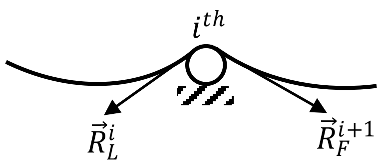

Fig. 3. FBD of a roller support of an MSCC system.

Converging to balanced support reactions is possible only by correct prediction. If not, correction step is needed for segment lengths of cable. Assuming quasilinear behavior of the system at the end of each predictive solution step, a relation between unstressed length adjustment Δl_u^j and corresponding change in the unbalanced reactions ΔT^i can be set up; where i and j denotes the support number. The relation is expressed in matrix form as follows.

$$\{\Delta T\} = [K] \cdot \{\Delta l_u\} \tag{9}$$

where, ΔT is a $(nx1)$ vector composed of ΔT^i, Δl_u is a (n x 1) vector composed of Δl_u^j, K is a $(n \text{ x } n)$ coefficient matrix composed of K_{ij}.

Coefficient matrix can be regarded as a tangential stiffness matrix in which K_{ij} represents the change in unbalanced reaction, δT^i, due to a change in unstressed length δl_u^j between cable segments j and $(j+1)$. The tangential stiffness matrix in Eq. (9) can be constructed column-by-column by adjusting the unstressed lengths of cable segments at support j by a small amount δl^j and calculating the resulting changes in the unbalanced reactions δT^i at all supports from the reanalysis of the SCC with the changed segment lengths. The j^{th} column of $[K]$ is obtained as.

$$\begin{Bmatrix} K_1 \\ \vdots \\ K_i \\ \vdots \\ K_n \end{Bmatrix} = \begin{Bmatrix} \delta T^1/\delta l^j \\ \vdots \\ \delta T^i/\delta l^j \\ \vdots \\ \delta T^n/\delta l^j \end{Bmatrix} \tag{10}$$

The objective is to balance the reactions at supports which is the static equilibrium condition. This is possible by applying required length adjustment for each segment. Length adjustments are achieved by solving Eq. (9). If the cable behavior were linear, the length adjustments $\{\delta l_u\}$ would eliminate the unbalanced reactions at intermediate supports and bring the cable system into true equilibrium. However, iterations are needed for the final equilibrium due to nonlinear behavior of cable. Newton-Raphson method is implemented for this predictive/corrective algorithm to reach the final equilibrium state.

2.3. Tension distribution method (TDM) (relaxation method)

Tension distribution method is a special form of the direct stiffness method. Being inception of analysis, TDM is inspired from the moment distribution method which is commonly used for the analysis of continuous beams. Relaxation method is the byname of this method. This additional name is given due to relaxation procedure at supports while balancing the reactions of cables at supports. In this context, this method is similar to DSM. The basic difference is the way of relaxation. While segment lengths are incremented for the whole system in DSM, slip amount is determined for two adjacent segments in TDM. Therefore, in the corrective stage, an influence (stiffness) coefficient is calculated at a selected joint first by introducing a virtual adjustment at the joint. Thus, the actual amount of adjustment required to eliminate the unbalanced reaction at the joint is determined based on this information. A cyclic procedure is needed for TDM, because elimination of unbalanced reactions (relaxation) is made for one support. Therefore, iterative cyclic calculations are carried out until the unbalanced reactions at the intermediate supports became negligibly small.

It is expected that; application of anticipated length adjustment Δl_u^i, for i^{th} roller support makes the tension difference ΔT^i, zero. Relation between Δl_u^i and ΔT^i is expressed in Eq. (11).

$$\Delta T^i = k^i \Delta l_u^i \tag{11}$$

The stiffness coefficient of the i^{th} roller support k^i, can be found by adjusting the unstressed lengths of adjacent segments by applying a small amount δl_u^i and calculating the resulting changes in the unbalanced reactions δT^i at that support as follows.

$$k^i = {\delta T^i}\Big/{\delta l_u^i} \tag{12}$$

In correction step of calculations, length adjustments can be calculated by the known tension difference ΔT^i and stiffness coefficient k^i from Eq. (11). It is not expected that; unbalanced reactions on each roller support to be zero in a single cycle of correction step due to nonlinear behavior of cable. Therefore, Newton-Raphson iterations are used to handle that nonlinearity.

In order to verify and prove the result of both methods, a benchmark cable system is created for MSCC system to point out the effect of cable motion on rollers.

3. Verification Cases

3.1. Case 1

Case 1 is a benchmark problem which is used by many researchers (Andreu et al. 2006; Jayaraman and Knudson 1962; Michalos and Birnstiel 1962; O'Brien and Francis 1964; Salehi et al. 2013; Thai and Kim 2011; Tibert 1999; Yang and Tsay 2007). A cable suspended by two fixed supports has its catenary shape as illustrated in Fig. 4. Initial properties of cable are given in Table 1. An external concentrated load is applied, and displacements of this node is determined. Results are comparatively shown in Table 2.

3.2. Case 2

Another SSC was defined by Peyrot and Goulois (1979) and used by researchers (Salehi et al. 2013; Yang and Tsay 2007). In this case, one end of the cable is fixed at a fixed position (0 m, 90 m) and the other end is moved starting from (0 m, 30 m) to (100 m, 30 m) as seen in Fig. 5. Initial properties of problem are given in Table 3. Reactions at the second end of the cable is compared as seen in Table 4.

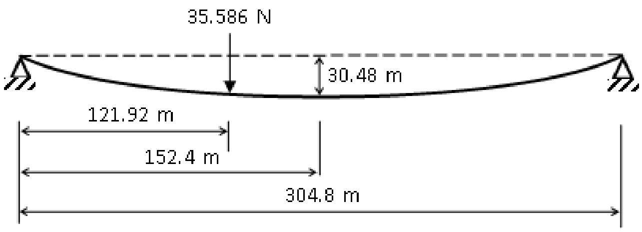

Fig. 4. SSC under concentrated load.

Table 1. Initial properties of Case 1.

Item	Data
Cable self-weight	46.12 N/m
Cross-sectional area	548.4 mm²
Elastic modulus	131 kN/mm²
Sag under self-weight at load point	29.262 m
Unstressed cable length 1-2	125.88 m
Unstressed cable length 2-3	186.85 m

Table 2. Comparison of results of Case 1.

Researcher	Vertical displacement (m)	Horizontal displacement (m)
Michalos and Birnstiel (1962)	-5.472	-0.845
Jayaraman and Knudson (1962)	-5.626	-0.859
Yang and Tsay (2007)	-5.625	-0.859
Thai and Kim (2011)	-5.626	-0.859
Andreu et al. (2006)	-5.626	-0.860
O'Brien and Francis (1964)	-5.627	-0.860
Tibert (1999)	-5.626	-0.859
Salehi et al. (2013)	-5.592	-0.855
SSC Solution	-5.626	-0.859

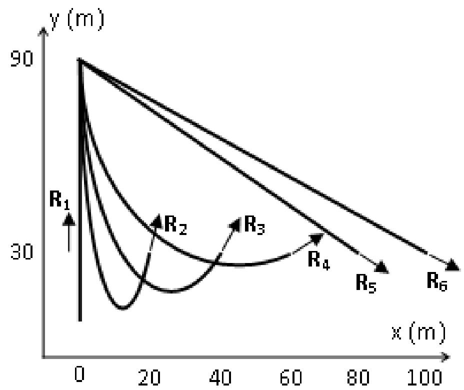

Fig. 5. SSC configuration of Case 2.

Table 3. Initial properties of Case 2.

Item	Data
Unstressed cable length	100 m
Cross-sectional area	1 m^2
Cable self-weight	1 N/m
Elastic modulus	3.0e7 N/mm^2
Thermal expansion coefficient	0.65e-5 1/°K
Thermal change	100°K

3.3. Case 3

In order to configure a multi segment continuous cable, a continuous cable is fixed at its ends ((0,0) and (300,0)) and supported by two rollers ((100,0) and (200,0)). All supports are positioned at the same elevation to point out the load effect on change in segmental lengths. Total cable length is selected longer than the total direct distance between supports, hereby a slack cable is achieved. Initially, the cable is loaded under its self-weight and a distributed load which is the ten times unit cable weight. This distributed load is applied on mid one fifth portion of whole cable. Initial properties of system are given in Table 5 and initial equilibrium state under self-weight and external distributed load is given in Fig. 6.

In this state, cable is almost linear at the first and the third segment. Theoretically, cable lengths at these segments should be equal due to symmetry. Taking maximum error as 0.1 mm and 0.1 N, cable lengths for each segment are given for finite element numbers in Fig. 7. As seen, almost same cable lengths are achieved for a small number of elements. In addition, maximum displacements at the midpoint of mid-segment are given for increasing number of elements in Fig. 8.

Table 4. The reactions at the second end of the cable.

Researchers	Peyrot and Goulois (1979)		Yang and Tsay (2007)		Salehi et al. (2013)		SSC Solution	
Reactions	x	y	x	y	x	y	x	y
R1	0.00	20.02	0.01	20.02	0.01	19.99	0.0	20.02
R2	3.061	19.93	3.061	19.93	3.090	19.83	3.061	19.942
R3	9.172	19.24	9.172	19.24	9.16	19.14	9.172	19.252
R4	22.15	15.73	22.15	15.73	22.11	15.63	22.146	15.744
R5	504.0	-328	504.1	-328.9	504.48	-329.4	504.102	-328.859
R6	4.17e6	2.511e6	42.56e6	-25.53e6	42.56e6	-25.55e6	42.58e6	-25.55e6

Table 5. Initial properties of Case 3.

Item	Data
Total unstressed cable length	315 m
Cross-sectional area	7.854e-5 m^2
Cable self-weight	6.0482 N/m
External load	60.482 N/m
Elastic modulus	200e9 N/m^2
Thermal expansion coefficient	1.2e-5 1/C

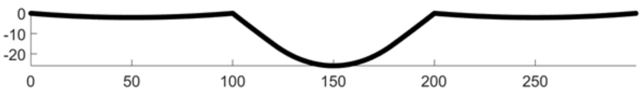

Fig. 6. Initial equilibrium state of Case 3.

In this state, cable is almost linear at the first and the third segment. Theoretically, cable lengths at these segments should be equal due to symmetry. Taking maximum error as 0.1 mm and 0.1 N, cable lengths for each segment are given for finite element numbers in Fig. 7. As seen, almost same cable lengths are achieved for a small number of elements. In addition, maximum displacements at the midpoint of mid-segment are given for increasing number of elements in Fig. 8.

Additional point loads are applied to see the changes in cable lengths of segments and the displacement of mid-segment. These point loads are applied to left segment and right segment. Amounts of point loads are 5080.488 N and 2540.488 N, respectively. Locations of forces on cable are specified as cable length which are 63 m and 252 m measured from left support of MSCC system. Final configuration of MSCC is given in Fig. 9 for 3000 finite elements and predefined precisions. Changes on MSCC system solved by TDM, DSM and ANSYS (a commercial computer program) are given in Tables 6-8, respectively. Solution times does not exceed a several minutes via a standard laptop computer. Solution time depends on mostly the cable slackness which decreases its stability. In ANSYS analysis, LINK10 element is used to model the cable. Besides, CONTA175 and TARGE169 elements are used for modelling of contact between the roller and the cable.

Table 6. Coordinates of nodes having maximum vertical displacement solved by TDM.

Segments	Initial state		Final state	
	X (m)	Y (m)	X (m)	Y (m)
Left segment	50	-2.0640	59.0275	-22.2794
Mid-segment	149.8866	-25.8670	148.6932	-11.5097
Right segment	250	-2.0798	237.8608	-10.4961

Table 7. Coordinates of nodes having maximum vertical displacement solved by DSM.

Segments	Initial state		Final state	
	X (m)	Y (m)	X (m)	Y (m)
Left segment	50	-2.0723	59.0277	-22.2806
Mid-segment	149.8866	-25.8670	148.6931	-11.5098
Right segment	250	-2.0798	237.8608	-10.4961

Table 8. Coordinates of nodes having maximum vertical displacement solved by ANSYS.

Segments	Initial state		Final state	
	X (m)	Y (m)	X (m)	Y (m)
Left segment	50	-2.0712	58.9193	-22.2874
Mid-segment	149.9024	-25.9248	148.6820	-11.5099
Right segment	250	-2.0768	237.7980	-10.4932

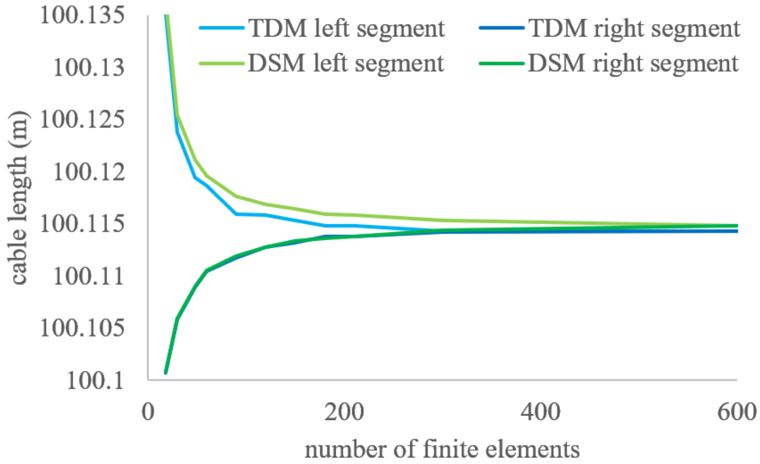

Fig. 7. Segment cable lengths vs. number of finite elements.

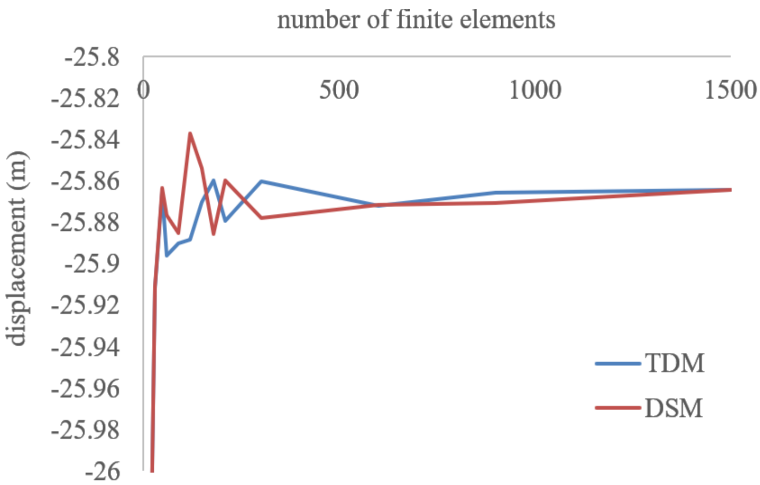

Fig. 8. Maximum displacement vs. number of finite elements.

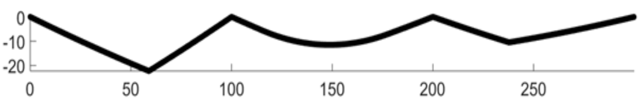

Fig. 9. Final equilibrium state of Case 3.

4. Conclusions

Multi-segment continuous cable (MSCC) has different behavior from single segment cable (SSC). A cable having constant length is fixed at its ends in SSC systems. In contrast, a cable having constant length is fixed at ends and supported by stationary and frictionless roller supports between ends in MSCC systems. Therefore, cable length of each segment is not constant for MSCC; it can change by the change of loading conditions as seen in the verification cases. This length change will change the resultant forces on cable and supports.

In this study, a novel solution approach is proposed for MSCC systems. Two methods are proposed; direct stiffness method (DSM) and tension distribution method (TDM) (relaxation method). DSM is imitated from the inherent motion of cable on roller supports and based on the stress continuity on the continuous cable. TDM is inspired from the moment distribution method, which has been used for continuous beam solutions.

DSM calculates length adjustment for the entire system consisting all segments which yields stress continuity through the cable. Length adjustments are calculated and applied for all roller supports. Nevertheless, stress continuity does not yield in one calculation phase. TDM calculates the length adjustment for two adjacent segments. Length adjustments are calculated and applied for each roller support, thus one cycle of calculations is fulfilled. Nevertheless, stress continuity does not yield in one cycle due to nonlinear behavior of cable. Newton-Raphson technique is used for both methods to overcome the nonlinearity.

Although DSM and TDM run in a similar manner, there are some differences. Those differences are due to the behavior of methods. DSM considers the circumstances on each segment while calculating the segment length adjustments. In contrast, TDM adjust two adjacent segment lengths by assuming other segments ineffective. Thus, those behaviors of methods give some advantages and disadvantages which are mainly related with the computational cost of the methods. DSM loses its effectiveness and speed for cable systems having many segments. In contrast, speed (not solution time) of TDM does not depend on the number of roller supports. Consequently, selection of method should be made accordingly, which will affect the solution time. Nevertheless, verification results show that both methods are effective and accurate methods for MSCC systems.

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Conflict of Interest

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