Barış Yıldız

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PRICING STRATEGIES UNDER PRICE PROTECTION, MID-LIFE RETURNS AND END-OF-LIFE RETURNS

A THESIS
SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING
AND THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE OF ABDULLAH GUL UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
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SCIENTIFIC ETHICS COMPLIANCE

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Ph.D. thesis titled Pricing Strategies under Price Protection, Mid-life Returns and End-of-life Returns has been prepared in accordance with the Thesis Writing Guidelines of the Abdullah Gül University, Graduate School of Engineering & Science.

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ACCEPTANCE AND APPROVAL

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ABSTRACT

PRICING STRATEGIES UNDER PRICE PROTECTION, MID-LIFE RETURNS AND END-OF-LIFE RETURNS

Barış Yıldız

Ph.D. in Industrial Engineering

Advisor: Asst. Prof. Dr. Muhammed Sütçü

December 2022

In this thesis, we examine a selling environment where a manufacturer-controlled retailer and an independent retailer sell a slow-moving *A* item. The manufacturer offers the independent retailer price protection against reductions in the wholesale price. The price set by the independent retailer is assumed to be determined by Retail Fixed Markdown (RFM) policy. The manufacturer adopts a periodic-review pricing strategy and each retailer observes price-dependent stochastic demand. We employ Multinomial Logit (MNL) models to forecast customers' preferences based on retail prices. We construct stochastic programming models to determine the manufacturer's pricing strategy in the presence of four distinct price commitment contracts which differ in the supplementary privileges combined with price protection. We also propose a variant Stochastic Dual Dynamic Programming (SDDP) algorithm to determine the manufacturer's approximately optimal pricing strategy by getting around three curses of dimensionality. We observe the impact of critically important contractual parameters on the price, the market shares and the expected true profits. We also evaluate the performance of the proposed algorithm and compare the price commitment contracts in terms of the contractual parameters for which it is crucial to choose a compromise value to ensure high enough profitability for both retailers.

Keywords: Price Commitment, Return Policies, Dynamic Pricing, Stochastic Dual Programming

ÖZET

FİYAT KORUMASI, DÖNEM ORTASI VE DÖNEM SONU GERİ ÖDEMESİ ALTINDA ÜCRETLENDİRME STRATEJİLERİ

Barış Yıldız
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Aralık 2022

Bu tezde, üretici tarafından yönetilen bir parekendecinin ve bağımsız bir parekendecinin görece daha az talep gören ve modası hızlı geçen *A* tipi bir ürünü sattıkları bir satış ortamını inceliyoruz. Üretici, bağımsız parekendeciye toptan satış fiyatındaki düşüşlere karşı fiyat koruması sağlıyor. Bağımsız parekendecinin tüketiciye sunduğu fiyatın Sabit Parekende İndirimi politikasıyla belirlendiği varsayılıyor. Üretici periyodik güncellemeli fiyatlandırma stratejisini benimsiyor ve her parekendeci fiyata bağlı olasılıksal talep alıyor. Müşterilerin satın alım tercihlerini toptan satış fiyatlarını değişken olarak alan Katlıterimli Logit modellerini kullanarak tahmin ediyoruz. Fiyat korumasıyla birlikte sunulan ek ayrıcalıklara göre değişkenlik gösteren birbirinden farklı dört fiyat yükümlülük sözleşmesinin varlığında üreticinin optimal fiyatlandırma stratejisini belirlemek için, olasılıksal programlama modellerini kuruyoruz. Üreticinin boyutluluğun üç lanetini bertaraf ederek yaklaşık olarak optimal olan bir fiyatlandırma stratejisi belirleyebilmesi için, Olasılıksal İkili Dinamik Programlama algoritmasının başka bir versiyonunu sunuyoruz. Kritik olarak önemli sözleşme parametrelerinin aldıkları değerlerin yaklaşık olarak optimal fiyat, pazar payları ve ortalama gerçek karlar üzerine olan etkisini gözlemliyoruz. Ayrıca sunulan algoritmanın performansını değerlendiriyoruz ve her iki parekendeci için yeterince karlılık sağlayabilmek adına bir uzlaşma değerinin seçilmesinin çok önemli olduğu sözleşme parametreleri bakımından fiyat yükümlülük sözleşmelerini karşılaştırıyoruz.

Anahtar kelimeler: Fiyat Yükümlülüğü, İade Politikaları, Dinamik Fiiyatlandırma, Olasılıksal İkili Programlama

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Last but not least, I want to strongly thank my parents for having persevering faith in me even when I complained of the difficulties that beleaguered me and despaired of handling them.

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LIST OF ABBREVIATIONS

LD	Lagrangian Dual
MNL	Multinomial Logit
RFM	Retail Fixed Markdown
SAA	Sample Average Approximation
SDDP	Stochastic Dual Dynamic Programming

Chapter 1

Introduction

In high-tech industry, customers tend to purchase technologically advanced brand new products or the improved models of the products they already have. Lee, Padmanabhan, Taylor and Wang [1] state that in the personal computer industry, products face rapid obsolescence that gives rise to slumps in prices throughout their life cycles so sellers are confronted with high demand uncertainty. This tendency compels manufacturers to make some changes in their product mixes. With the development, production and introduction of some brand new products, the old products are offered at discounted prices to the customers that have relatively low budgets.

As manufacturers can sell their products to the end customer via its own retailers, they may also prefer collaborating with some retailers in order to reach much more customers in the market. For this purpose, manufacturers should offer some privileges to entice retailers into keeping the inventory of their products. Especially, external retailers want to be protected against sudden drops in the wholesale prices at which they purchase products. Sourirajan, Kapuscinski and Ettl [2] state that price protection is intended to induce distributors and retailers to keep adequate inventory by protecting them against sudden drops in the price of the corresponding product. Manufacturers offer a price protection contract by which they assure retailers that they are committed to reimbursing retailers the amount of reduction in wholesale prices per product for the inventory retailers have in stock.

Manufacturers might also offer some other privileges than price protection. One of these privileges is mid-life returns. If a manufacturer grants a retailer the opportunity of returning some of its inventory at any time of the selling horizon, then it also agrees to refund the retailer some money per returned product. Likewise, if the product in question will be withdrawn from the market after a selling horizon of a predetermined length, retailers can also be allowed to return their remaining on-hand inventory at the end of that selling horizon in exchange for some refund. This is called end-of-life returns.

In the literature, there is a variety of research papers where the effects of different price commitment policies on channel coordination in different selling environments are analyzed in a two-period case where there exist one manufacturer and one retailer [1, 3-6]. Researchers present the conditions that have to be satisfied to ensure channel coordination for some policies which are capable of coordinating a supply chain. These policies are price protection, mid-life returns and end-of-life returns. There also exist research papers in which some supplementary policies are evaluated in conjunction with price protection, mid-life returns and end-of-life return in a two period case where there exist one manufacturer and one retailer [7, 8].

These studies mostly focus on which inventory replenishment or return policy is optimal and researchers aim to determine the optimal policy parameters. In this kind of research papers, the retail price and the wholesale price set in each period of the selling horizon are assumed to be fixed and researchers employ demand distributions that are estimated based on fixed retail prices. However, it is also intriguing whether or not the evaluated price commitment policies and return policies are capable of coordinating a supply chain and providing a win-win outcome in an environment where retail prices are decision variables and they have an influence on demand distributions. For that purpose, we only focus on pricing strategies in this study by excluding channel coordination and win-win outcome concerns and reckoning with price-dependent consumer behaviors in order to lay a foundation for further researches on channel coordination in selling environments where responsive pricing strategy is adopted instead of pre-announced pricing strategy.

In the literature, there are research papers in which pricing and inventory control decisions are studied and analyzed simultaneously. Chen, Chen, Keblis and Lee [9] (2019) study a selling environment where a deteriorating product that is assumed to have a short lifecycle is sold throughout a finite selling horizon of multiple periods. They assume deterministic, stock level-dependent, time-varying and price-dependent demand for the product and develop an algorithm to determine a profit-maximizing replenishment and pricing policy.

Ghoreishi, Mirzazadeh, Weber and Nakhai-Kamalabadi [10] build an Economic Order Quantity model for non-instantaneous deteriorating items for which inflation- and selling price-dependent demand is observed by allowing partial backlogging and customer returns. They propose an efficient algorithm intended to simultaneously optimize the selling price, the length of the replenishment cycle and the length of time when shortage does not occur.

Mishra [11] builds a model for a deteriorating item for which stock level- and selling price-dependent demand is observed by assuming Weibull deterioration and partial backlogging. The author proposes a simple algorithm designed to simultaneously optimize the selling price, the replenishment schedule and the order quantity with the purpose of maximizing the total profit.

Nagaraju, Rao and Narayanan [12] focus on both a centralized and a decentralized three-echelon inventory system consisting of a manufacturer, a distributor and a retailer. The author assumes that selling price-dependent deterministic demand is observed for the product and the selling price is a function of the replenishment quantity with dependence factor. Some managerial insights on the optimal selection of replenishment quantity and shipment frequency are set forth in the paper.

In another research paper, pricing and inventory decisions for a two-echelon inventory system are discussed by assuming nonlinear price-dependent demand [13]. Agi and Soni [14] build a deterministic model and propose an algorithm meant for the simultaneous optimization of the selling price, the cycle length, the order quantity and the end-of-cycle inventory for a perishable product. The authors assume that demand depends on the selling price, the current inventory level and the freshness condition.

If a manufacturer and the retailers with which it collaborates sell a product to the end customer, then there exists a natural competition. Therefore, unlike the previous studies, the impact of the retail prices on consumers' purchasing behavior has to be taken into account, as well. That is, it is not convenient to employ even price-dependent demand functions. Instead, price-dependent stochastic demand distributions should be employed. As a starting point for such analyses, we commit ourselves to examining the impact of different price commitment contracts on the optimal retail price and the actors' profits. However, we do not deal with the determination of the optimal inventory replenishment policy and the optimal return policy. Each commitment contract discussed in this study includes a distinct combination of price protection, mid-life return opportunities, end-of-life return opportunity and a special discount policy.

In this study, a manufacturer-controlled retailer and an independent retailer sell a slow-moving A item throughout a finite selling horizon which is partitioned into periods. The manufacturer sets the retail price at the beginning of each period and the manufacturer-controlled retailer sells the product to the end customer at that price. The manufacturer sets the wholesale price that it asks the independent retailer to pay per product in case of replenishment by discounting the retail price offered by the manufacturer-controlled retailer. The independent retailer offers the end customer a discounted price in accordance with Retail Fixed Markdown (RFM) policy. We assume a non-increasing price environment resulting from the depreciation of the product over time. We study four different price commitment contracts and in all the contracts discussed, the manufacturer offers the independent retailer price protection. As per these contracts, the manufacturer also allows the independent retailer to return the entire unsold inventory at the end of the selling horizon. The distinction between the contracts arises from whether mid-life return opportunities and a special discount policy are included or not and whether they are present individually or concurrently if either of them is included. Both retailers are allowed to place a replenishment order at the beginning of each period. In this study, we make an assumption on the inventory replenishment policies that the retailers follow throughout the selling horizon. We also employ price-dependent stochastic demand distributions by taking the influence of the retail prices on consumers' valuations about the retailers into consideration.

The manufacturer aims to develop a profit-maximizing pricing strategy. The optimal pricing strategy can be determined by modeling the problem through dynamic programming (DP) approach. However, the state space, the decision space and the random event space are infinite so the DP model is plagued by the three curses of dimensionality defined in Powell [15]. Therefore, our objective is to propose a suitable method meant to determine an approximately optimal pricing strategy for the manufacturer and to analyze the impact of different price commitment contracts on this approximately optimal pricing strategy and the manufacturer's and the independent retailer's true expected total profits given that strategy.

Stochastic Dual Dynamic Programming (SDDP) algorithm first proposed by Pereira and Pinto [16] and analyzed further in Chen and Powell [17], Donohue and Birge [18], Linowsky and Philpott [19] and Philpott and Guan [20] for finite random data process skillfully deals with the estimation of the post-decision profit-to-go functions by deriving an upper bound on each iteration and it has a legitimate stopping criterion. However, the random data process is assumed to be finite in these papers. As in our case, the random data process is generally infinite in most real-life applications.

For this reason, Shapiro [21] discusses how the SDDP algorithm can be implemented when the random data process is infinite. Since the algorithm is also well-suited to the cases in which random data process is infinite, this algorithm is applicable to our case. For that reason, we model our problem through a stochastic programming approach.

SDDP algorithm is a simulation-oriented method the initialization of which necessitates the generation of a number of realizations from random event distributions. However, demand distributions that depend on the decision variable preclude the direct generation of demand realizations in our problem. For this reason, we propose a modified version of the SDDP algorithm to determine the manufacturer’s approximately optimal pricing strategy. By doing so, we also want to shed light on how the problems afflicted by three curses of dimensionality in which random event distribution depends on the decision variable can be dealt with in case the parametric expression of the optimal solution is not possible.

The organization of the thesis is as follows; in Chapter 2, we provide the full definition of the problem where the manufacturer offers the independent retailer protection and end-of-life return opportunity, present the mathematical model constructed to determine the manufacturer’s optimal pricing strategy, explain the modified SDDP algorithm conceived to propose the manufacturer an approximately optimal pricing strategy and present the results of the numerical experiment intended to observe how the changes in some contractual parameters impact the approximate optimal price, the retailers’ market shares and their true expected total net profits. In Chapter 3, we discuss the price commitment contract in which mid-life return opportunities are combined with price protection and end-of-life return opportunity. In Chapter 4, we discuss another price commitment contract where mid-life return opportunities are substituted by a special discount policy. In Chapter 5, we discuss the price commitment contract where both mid-life return opportunities and a special discount policy are present in conjunction with price protection and end-of-life return opportunity. In Chapter 6, we share the conclusions we draw and discuss future research opportunities.

Chapter 2

Periodic-review Approximately Optimal Pricing in the Presence of Price Protection and End-of-life Return Opportunity

In this chapter, firstly, the problem definition is provided by defining the boundaries of the research built on some assumptions. Secondly, the stochastic programming model to be solved to determine the manufacturer's optimal pricing strategy is presented. Thirdly, the variant SDDP algorithm proposed to circumvent the three curses of dimensionality and to determine the manufacturer's approximately optimal pricing strategy is explained. Finally, the results of the numerical experiments carried out to observe how the changes in the four contractual parameters playing a significant role in the manufacturer's pricing decisions impact the approximate optimal price, the retailers' market shares and their true expected total net profits are presented.

2.1 Problem Definition

In this problem, a manufacturer-controlled retailer and an independent retailer sell a slow-moving *A* item to the end customer throughout a finite selling horizon. The manufacturer is responsible for the production and the delivery of the corresponding product to the retailers. At the beginning of each period, the manufacturer determines the retail price at which the manufacturer-controlled retailer will sell the product to the end customer. A non-increasing price environment is assumed by neglecting some extraordinary external factors such as erratically changing foreign currency parities, inflation rate, interest rate etc. that might have an impact on the manufacturing cost of the product. That is, the product keeps depreciating over time so the manufacturer does not increase the retail price throughout the selling horizon. In purpose for enticing the independent retailer into keeping the inventory of the product during the selling horizon, the manufacturer offers the independent retailer price protection in case of a reduction in the retail price. The manufacturer is committed to reimbursing the independent retailer for a fixed proportion of the independent retailer's on-hand inventory whenever it decreases the retail price. From the second period on, reimbursement should be fulfilled at the beginning of each period in which the manufacturer decides to sell the product at a lower price.

Both of the retailers are allowed to make replenishment at the beginning of each period after the manufacturer sets the retail price. The necessary lead time for the production and the delivery of replenishment orders is assumed to be negligibly short. Since the purpose of this study is to determine the manufacturer's optimal pricing strategy instead of optimal replenishment policies, an assumption is made about the replenishment policies that these retailers follow throughout the selling horizon. This assumption is inspired by the research papers that study channel-coordinating replenishment policies in case of fixed retail and wholesale prices. By the assumption, both of the retailers follow order-up-to inventory replenishment policy (R, S) which is proven to coordinate supply chains in many problem settings discussed in Lee et al. [1], Lee and Rhee [7] and Liu, Fry, Qin and Raturi [22]. The independent retailer's order-up-to level is negotiable because retailers are inclined to order in large batches in presence of price protection and end-of-life return opportunity. Replenishment policy parameters are fixed in the price commitment contract and they cannot be changed during the selling horizon. It is assumed that both retailers have no stock before they make replenishment at the beginning of the selling horizon.

The manufacturer enables the independent retailer to purchase products at a discounted price. After the manufacturer sets the retail price in a given period, the wholesale price is determined by discounting the retail price. The discount rate is assumed to be fixed over time. Both of the retailers are allowed to backorder a fixed maximum allowable amount of demand in each period except the last one in case they observe excess demand. The retailers determine the size of their replenishment orders by reckoning with backordered demand. From the second period on, the backordered demand is satisfied after the replenishment order is delivered at the beginning of each period. The retailers offer their customers a special discount for the backordered demand. That discount is applied on the price that customers would have paid if the product had been available in the previous period. No lost sales cost is incurred for the other customers turned down in a stockout.

After the manufacturer sets the retail price in a given period, the retail price at which the independent retailer will sell the product to the end customer is determined by Retail Fixed Markdown (RFM) policy. That price is computed by marking down the retail price set by the manufacturer by a fixed rate. The objective with the application of RFM policy is to lure customers from a lower-income segment so as to raise revenue. The markdown rate has to be less than the discount rate for the independent retailer's profitability throughout the selling horizon.

Since these retailers are natural competitors in the market, the retail prices have an influence on their demand distributions. Poisson distribution is a perfect fit to model demand behavior for a slow-moving *A* item as proposed by Silver, Pyke and Peterson [23]. Therefore, the distribution of the number of potential customers in each period is assumed to be Poisson with an estimated and known mean value. Potential customers either purchase the product from either retailer or leave the market. The retailers' market shares given the retail prices can be estimated by employing a suitable choice model. The demand observed by each retailer in a given period is also Poisson distributed since a Poisson process can decompose into Poisson sub-processes. The mean demand observed by a given retailer in a given period equals the mean number of potential customers multiplied by the retailer's market share.

The independent retailer is allowed to salvage its entire remaining inventory by returning the unsold products to the manufacturer at the end of the selling horizon in return for a refund. The manufacturer also salvages its leftover inventory and the products returned by the independent retailer.

This problem is modeled through stochastic programming approach to maximize the manufacturer's expected total profit. The stochastic programming model is presented in the following section.

2.2 Model

In this section, the stochastic programming model constructed to maximize the manufacturer's expected total profit is presented. The length of the selling horizon is assumed to be N periods. The notation used throughout the section is shown in Table 2.1 presented below.

Table 2.1 Notation

$\boldsymbol{R}$	Inventory holding cost per dollar per period
$\boldsymbol{\theta}$	Discount rate offered to customers for backordered demand
$\boldsymbol{\beta}$	Discount rate offered to the independent retailer for replenishment orders
$\boldsymbol{\alpha}$	Reimbursement rate
$\boldsymbol{c_t}$	Production cost per product in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t^m}$	Demand observed by the manufacturer-controlled retailer (*m*) in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t^r}$	Demand observed by the independent retailer (*r*) in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t}$	Ordered pair (D_t^m, D_t^r) of demands observed by retailers in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{N_t^m}$	Maximum allowable amount of demand that can be backordered by manufacturer-controlled retailer (*m*) in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{N_t^r}$	Maximum allowable amount of demand that can be backordered by independent retailer (*r*) in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{S_t^m}$	Order-up-to level that the manufacturer-controlled retailer (*m*) needs to place a replenishment order if inventory level goes below at the beginning of period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{S_t^r}$	Order-up-to level that the independent retailer (*r*) needs to place a replenishment order if inventory level goes below at the beginning of period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{I_t^m}$	Manufacturer-controlled retailer (*m*)'s on-hand stock right before observing demand in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{I_t^r}$	Independent retailer (*r*)'s on-hand stock right before observing demand in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{w_m}$	Salvage value per product of manufacturer-controlled retailer (*m*)'s unsold inventory at the end of the selling horizon
$\boldsymbol{w_r}$	Refund per product returned by the independent retailer (*r*) at the end of the selling horizon
$\boldsymbol{p_t}$	Retail price set by the manufacturer in period *t* $\forall t \in \{1,2,\dots N\}$

In the first period, the manufacturer sets the retail price and then the manufacturer-controlled retailer and the independent retailer make replenishment to raise their inventory levels to their order-up-to levels S_1^m and S_1^r, respectively. As a result, the manufacturer makes some post-decision profit by selling some products to the independent retailer. After replenishments, the manufacturer generates some extra post-decision revenue by selling the product to the end customer in the first period and selling some products to the independent retailer in case of a replenishment order at the beginning of the second period. Therefore, the retailers' inventory levels after replenishments and the demand observed by each retailer in the first period are determinants of the manufacturer's pricing decision in the second period. For that reason, a post-decision profit-to-go function $(\varphi_1(S_1^m, S_1^r, p_1))$ of the retailers' inventory levels (S_1^m and S_1^r) after replenishments and the price (p_1) set by the manufacturer in the first period connects the first-stage problem to the second-stage problem. The post-decision profit-to-go function of a given period returns the optimal expected total profit the manufacturer makes from after it sets the retail price and the retailers make replenishments till the end of the selling horizon given the inventory levels after replenishments and the retail price set for the given period. The post-decision profit-to-go function of the first-stage problem is exactly the expected value of the pre-decision profit-to-go function ($Q_2(S_1^m, S_1^r, p_1, D_1)$) of the second-stage problem over the ordered pair (D_1) of demands observed by the retailers in the first period. The model that has to be solved in the first period is as follows:

$$\max_{p_1 \in A_1} -S_1^m . c_1 + \left((1-\beta). p_1 - c_1\right). S_1^r + \varphi_1(S_1^m, S_1^r, p_1), \tag{2.1}$$

where

$$A_1 = \{p_1 \in \mathbb{R}: p_1 \geq 0\}, \tag{2.2}$$

$$\varphi_1(S_1^m, S_1^r, p_1) = \mathbb{E}_{D_1}[Q_2(S_1^m, S_1^r, p_1, D_1)]. \tag{2.3}$$

At the beginning of a given intermediate period *t*, the manufacturer incurs inventory holding cost per product carried over from period *t-1*, earns some money by selling products to the independent retailer in case the independent retailer places a replenishment order, and incurs some production cost stemming from the retailers' replenishment orders if there is any. The retailers' replenishment orders also cover the products backordered in the period *t-1*. Since there is a non-increasing price environment, if the manufacturer changes the price of the product, it reimburses the independent retailer for a fixed proportion of the unsold inventory carried over by the independent retailer from period *t-1* in compliance with the price commitment contract. Therefore, there exists a reimbursement cost term in the objective function. The objective function also contains the earnings from selling products in period *t-1*. Furthermore, we know that the retailers' inventory levels at the beginning of the period have an influence on the manufacturer's pricing decision. For that reason, we have to solve the model of period *t* for the retailers' all possible inventory levels before they observe demand in period *t-1* and all possible amounts of demand that they can observe in period *t-1*. The possible inventory levels that a given retailer can have at the beginning of period *t-1* range between the given retailer's order-up-to level in period *t-1* and the maximum of its order-up-to levels till period *t-1*. Then, the model that has to be solved for a given intermediate period *t* ($t \neq 1$) is as follows:

$$
\begin{aligned}
Q_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}) = \max_{p_t \in A_t} & - max\{min\{D_{t-1}^m - I_{t-1}^m, N_{t-1}^m\}, 0\}\,.\,\theta.\,p_{t-1} \\
& + min\{D_{t-1}^m, I_{t-1}^m + N_{t-1}^m\}.\,p_{t-1} \\
& - max\{I_{t-1}^m - D_{t-1}^m, 0\}.\,c_{t-1}.\,r \\
& - max\{S_t^m - I_{t-1}^m + D_{t-1}^m, 0\}.\,c_t \\
& + max\{D_{t-1}^m - I_{t-1}^m - N_{t-1}^m, 0\}.\,c_t \\
& + max\{S_t^r - I_{t-1}^r + D_{t-1}^r, 0\}.\,\big((1-\beta).\,p_t - c_t\big) \\
& - max\{D_{t-1}^r - I_{t-1}^r - N_{t-1}^r, 0\}.\,\big((1-\beta).\,p_t - c_t\big) \\
& - max\{I_{t-1}^r - D_{t-1}^r, 0\}.\,\alpha.\,(p_{t-1} - p_t) \\
& + \varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m)\,, IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t), \qquad (2.4)
\end{aligned}
$$

where

$$A_t = \{p_t \in \mathbb{R}: p_t \le p_{t-1}, p_t \ge 0\}, \qquad (2.5)$$

$$\varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m)\,, IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t) = \mathbb{E}_{D_t}[Q_{t+1}(IL_t^m(I_{t-1}^m, D_{t-1}^m)\,, IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t, D_t)], \qquad (2.6)$$

$$IL_t^m(I_{t-1}^m, D_{t-1}^m) = max\{S_t^m, I_{t-1}^m - D_{t-1}^m\}, \qquad (2.7)$$

$$IL_t^r(I_{t-1}^r, D_{t-1}^r) = max\{S_t^r,\, I_{t-1}^r - D_{t-1}^r\}. \qquad (2.8)$$

The functions shown in Equation (2.7) and Equation (2.8) return the manufacturer-controlled retailer's and the independent retailer's post-replenishment inventory levels at the beginning of the period. At the end of the selling horizon, the independent retailer returns its unsold inventory to the manufacturer and gets refunded. The manufacturer salvages the manufacturer-controlled retailer's unsold inventory and the products returned by the independent retailer. Then, the profit function of the dummy period *N+1* given the retailers' inventory levels before observing demand in period *N*, demand observed by the retailers in period *N* and the price set by the manufacturer in period *N* is as follows:

$$
\begin{aligned}
Q_{N+1}(I_N^m, I_N^r, p_N, D_N) = & - max\{I_N^m - D_N^m, 0\}.\,c_N.\,r + D_N^m.\,p_N - max\{D_N^m - I_N^m, 0\}.\,p_N \\
& + max\{I_N^m - D_N^m, 0\}.\,w_m + max\{I_N^r - D_N^r, 0\}.\,(w_m - w_r). \qquad (2.9)
\end{aligned}
$$

In this model, the state space, the decision space and the random event space are infinite. For that reason, only if closed-form expressions can be obtained for the post-decision profit-to-go functions, the exact optimal price that the manufacturer should set to maximize its expected total profit can be determined. However, it is not possible to solve the optimization problems parametrically. For that reason, we propose a variant SDDP algorithm that can be implemented to determine the approximately optimal price in the following section.

2.3 Methodology

The multi-stage decision-making is a very troublesome task if a sort of randomness should be taken into account and the random event is identified by a continuous random variable which can inherently take on infinitely many values. However, even the discreteness of the randomness does not facilitate the process if the support of the corresponding discrete random variable is infinite. The hardness triggered by the infinite support of a random event arises from infinite random event space which is one of the curses of dimensionality elaborated on by Powell [15].

Furthermore, even if the random variable characterizing a random event takes on finitely many values, the existence of an infinite decision space most of the time renders it impossible to keep track of the evolution of optimal actions over time. In that case, the attainment of the exact optimal action at each stage necessitates deriving the closed-form expression of the profit-to-go function of the following stage which provides the expected total profit made till the end of the horizon given the action at that specific time epoch. Under these circumstances, if there is no effective means of circumventing this trouble, the strategist is compelled to relinquish exact optimal actions and driven to seek out a way to obtain an approximate solution. In our model, we are up against such a trouble, as well. In compliance with the problem definition, the manufacturer is allowed to set a continuous price and the distribution of the random demand observed by the manufacturer-controlled retailer and the independent retailer shows a Poisson behavior. For this reason, we have to look for a methodology that supplies us with an implementable set of procedures leading to an approximate optimal pricing policy.

In the literature, some approximate dynamic programming algorithms are proposed to circumvent the difficulties that arise from the existence of three curses of dimensionality [15]. The fast convergence of these algorithms necessitate decent estimations of the value functions such as pre-decision profit-to-go functions, post-decision profit-to-go functions, Q-factors etc. However, the existence of a proper and reliable way of estimating the value functions is questionable. Furthermore, the optimization is done sequentially by starting from the first stage and generating a realization from the distribution of random event occurring between stages on each iteration. That is, a single state is visited at each stage on each iteration. Even if the demand distributions depend on the decision variable in our problem, sequential optimization helps us calculate the retailers' market shares given the optimal action through special market share functions and estimate the demand distributions based on those market shares at each stage. For that reason, these algorithms are adaptable to our problem. However, the applicability is also a very critical evaluation measure in the selection of a suitable adaptable methodology.

These algorithms entail updating the estimations of the value functions through a smoothing operation on each iteration by using the estimations of the previous iteration and the approximate optimal values of the current iteration. However, the concern about the smoothing operation is the selection of an appropriate step size. Moreover, these algorithms terminate when the number of iterations reaches the preset maximum value. However, the selection of that value is a little bit problematic in our case because the convergence is bound to require a very large number of iterations. Therefore, these algorithms are not applicable to our problem although they are adaptable because of the complicated structure of the problem.

Pereira and Pinto [16] proposes an algorithm called Stochastic Dual Dynamic Programming (SDDP) specialized in multi-stage decision-making. In various research papers such as Chen and Powell [17], Donohue and Birge [18], Linowsky and Philpott [19] and Philpott and Guan [20], this algorithm is extended and analyzed for the case where random data process is finite. However, the random data process is infinite in our study. As an extension to the earlier research, Shapiro [21] discusses the implementation of the SDDP algorithm in case of an infinite random data process. In Shapiro [21], it is assumed that the random data process is stage-wise independent implying that the probability of a given random realization at a given stage does not depend on the random realization observed in the previous stage. Moreover, the distributions of the random events occurring between two consecutive stages are assumed to be known in advance. That is, not only are the types of the distributions known in advance, but the parameters are also fixed. Another assumption is that the optimal decision at a specific stage depends on only the random realization observed right before and the action taken at the preceding stage. That is, the history of random realizations does not have an influence on the decision maker’s preference.

In our problem setting, the mean number of potential customers in the market in each period of the selling horizon is estimated in advance. As explained in Section 2.1, the mean demand observed by a given retailer in a given period depends on the retail price the manufacturer sets. This means that although we know that the distribution of the demand observed by each retailer is Poisson, the manufacturer’s pricing decision influences its mean. Furthermore, since the inventory levels of the retailers are determinants of how the manufacturer will react, the retailers' starting inventory levels at the beginning of each period have to be reckoned with. Although the actions taken by the manufacturer in two consecutive periods are dependent because of the non-increasing price assumption, fortunately, the random data process is still stage-wise independent given a feasible set of actions taken by the manufacturer from the beginning of the selling horizon till the end.

The differences in our model require the ideation of a new algorithm which is a variant of SDDP algorithm that is capable of handling the price-dependent infinite random event space. Through the variant SDDP algorithm, we will be able to analyze the case where the retail price set by the manufacturer in a given period has an influence on the demand distributions. Otherwise, the SDDP algorithm proposed and extended over time in the literature is not capable of dealing with this case. It will be explained below how the original SDDP algorithm proposed under some certain assumptions is adapted to the specifications of our decision-making process.

SDDP algorithm is proposed to solve sample average approximation problem (SAA) as explained in Shapiro [21]. The algorithm is executed iteratively and involves the consecutive implementations of two steps on each iteration. These steps are called backward step and forward step. The backward step is initialized by generating a number of realizations from the random event distributions in purpose for simulating the random behavior. Likewise, we have to generate a chosen number of Poisson demand realizations observed by the manufacturer-controlled retailer and the independent retailer in each period to build the SAA problem. Since the mean demand observed by a given retailer depends on the manufacturer's pricing decision, we cannot know the exact mean value in advance. For that reason, this fact obstructs the generation of ordered pairs of demand realizations for each period beforehand. However, we can simply generate realizations for the number of potential customers in the market for each period because the estimated mean aggregate demand is known in advance by problem definition. This implies that we can construct the SAA problem by substituting the expectation of the conditional expectation of the pre-decision profit-to-go function of the following period given the number of potential customers over all possible numbers of potential customers for the post-decision profit-to-go function of the current period in the objective function of the mathematical model of each period. For a given period t, this relation is as follows:

$$\varphi_t(PRIL_t^m, PRIL_t^r, p_t) = \mathbb{E}_{A_t}\left[\mathbb{E}_{D_t}\left[Q_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (D_t^m, D_t^r)\right) \setminus A_t = a\right]\right]. \quad (2.10)$$

In the equation shown above, $PRIL_t^m$ and $PRIL_t^r$ stand for the manufacturer-controlled retailer's and the independent retailer's inventory levels right after the replenishments at the beginning of the period *t*. If M_t realizations are generated for the number of potential customers observed in the market in a given period *t*, then the approximate post-decision profit-to-go function ($\tilde{\varphi}_t(PRIL_t^m, PRIL_t^r, p_t)$) of that period is expressed as follows:

$$\tilde{\varphi}_t(PRIL_t^m, PRIL_t^r, p_t) = \frac{1}{M_t} \cdot \sum_{k=1}^{M_t} \mathbb{E}_{D_t}\left[\tilde{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (D_t^m, D_t^r)\right) \setminus A_t = a_{t,k}\right]. \quad (2.11)$$

In Equation (2.11) shown above, $a_{t,k}$ stands for the *k*th realization generated for the number of potential customers observed in period *t*. The approximate pre-decision profit-to-go function ($\tilde{Q}_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1})$) of a given period *t* except the first period is characterized by the optimal value of the nonlinear SAA model shown below given the retailers' inventory levels (I_{t-1}^m and I_{t-1}^r) after replenishments at the beginning of period *t-1*, the retail price (p_{t-1}) of the product set by the manufacturer in period *t-1* and the ordered pair (D_{t-1}) of demands observed by the retailers in period *t-1*.

$$\begin{aligned}\tilde{Q}_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}) = \max_{F_t} \; & PRD_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}^m, D_{t-1}^r) + POD_t(I_{t-1}^r, p_t, D_{t-1}^r) \\ & + \tilde{\varphi}_t(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t), \end{aligned} \quad (2.12)$$

where

$$F_t = \left\{p_t \in \mathbb{R}: p_t \le p_{t-1},\ p_t \ge 0\right\}, \quad (2.13)$$

$$\begin{aligned}&\tilde{\varphi}_t(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t) = \\ &= \frac{1}{M_t} \cdot \sum_{k=1}^{M_t} \mathbb{E}_{D_t}\left[\tilde{Q}_{t+1}\left(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t, (D_t^m, D_t^r)\right) \setminus A_t = a_{t,k}\right],\end{aligned} \quad (2.14)$$

$$IL_t^m(I_{t-1}^m, D_{t-1}^m) = max\{S_t^m, I_{t-1}^m - D_{t-1}^m\}, \quad (2.15)$$

$$IL_t^r(I_{t-1}^r, D_{t-1}^r) = max\{S_t^r, I_{t-1}^r - D_{t-1}^r\}. \quad (2.16)$$

In the nonlinear model presented above, $PRD_t(.)$ and $POD_t(.)$ are the functional representations of the pre-decision profit made right before the pricing decision for period *t* and the post-decision profit made right after the pricing decision in the current period *t*, respectively. The SAA model to be solved to determine the approximate optimal price to be set in the first period of the selling horizon is as follows:

$$\max_{F_1} FPP(p_1) + \tilde{\varphi}_1(S_1^m, S_1^r, p_1), \tag{2.17}$$

where

$$F_1 = \{p_1 \in \mathbb{R}: p_1 \geq 0\}. \tag{2.18}$$

In the nonlinear model presented above, $FPP(.)$ stands for the functional representation of the profit made right after the replenishments at the beginning of the selling horizon. Despite the new formulation of the problem, it is still hard and most of the time impossible to derive a closed-form expression for the approximate post-decision profit-to-go functions. Therefore, we have to derive a function of the decision variable which returns an upper bound over the corresponding approximate post-decision profit-to-go function of each period. In this case, if we mean to derive an upper bound over the approximate post-decision profit-to-go function of a given period *t* given the retailers' inventory levels right after the replenishments at the beginning of the corresponding period, we have to derive an upper bound function for the conditional expectation of the approximate pre-decision profit-to-go function of the succeeding period *t+1* given each realization generated for the number of potential customers in period *t* at the beginning of the backward step and then take the sample average of these functions. This relation is as follows:

$$\tilde{\varphi}_t(PRIL_t^m, PRIL_t^r, p_t) \leq \bar{\varphi}_t(PRIL_t^m, PRIL_t^r, p_t), \tag{2.19}$$

where

$$\bar{\varphi}_t(PRIL_t^m, PRIL_t^r, p_t) = \frac{1}{M_t} \cdot \sum_{k=1}^{M_t} \overline{\mathbb{E}}_{D_t}\big[\tilde{Q}_{t+1}(PRIL_t^m, PRIL_t^r, p_t, D_t) \setminus A_t = a_{t,k}\big]. \tag{2.20}$$

Since these two retailers are assumed to follow order-up-to inventory replenishment policy, $PRIL_t^m$ ranges between S_t^m and the maximum of all the order-up-to levels from the beginning of the selling horizon to period *t*. The same applies to $PRIL_t^r$ implying that there are finitely many pairs of inventory levels for which we have to find an upper bound for the approximate post-decision profit-to-go function of period *t*.

Given the number of potential customers in the market in an arbitrary period *t*, the amount of demand observed by either retailer in that period is trinomially distributed. The probability that a potential customer prefers purchasing the product from a given retailer is that retailer's price-dependent market share by problem definition. Let

$MS_t^m(p_t)$, $MS_t^r(p_t)$ and $MS_t^o(p_t)$ stand for the manufacturer-controlled retailer's market share, the independent retailer's market share and the probability that a potential customer chooses no-purchase option given the price p_t set by the manufacturer in period *t*, respectively. Then, Equation (2.20) can be expanded as follows:

$$\bar{\varphi}_t(PRIL_t^m, PRIL_t^r, p_t) = \frac{1}{M_t} \cdot \sum_{k=1}^{M_t} \sum_{l=0}^{a_{t,k}} \sum_{m=0}^{a_{t,k}-l} \left(\bar{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (l,m)\right)\right. .$$

$$\left. \cdot \binom{a_{t,k}}{l} \cdot \binom{a_{t,k}-l}{m} \cdot (MS_t^m(p_t))^l \cdot (MS_t^r(p_t))^m \cdot (MS_t^o(p_t))^{a_{t,k}-l-m}, \right. \quad (2.21)$$

where

$$\tilde{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (l,m)\right) \leq \bar{Q}_{t+1}\left(PRIL_t^m, PRIL_t^r, p_t, (l,m)\right). \quad (2.22)$$

As can be seen in the equation presented above, we have to derive an upper bound function for the pre-decision profit-to-go function of the succeeding period *t+1* given the inventory levels after replenishments in period *t* and for each possible pair of demands observed by the retailers in period *t* given each realization generated for the number potential customers in the market in period *t*. Afterwards, we have to take the sample average of all the upper bound functions multiplied by the trinomial mass function over all the realizations generated for the number of potential customers in period *t* at the beginning of the backward step. The quality of the upper bound dramatically impacts how fast the algorithm converges. Therefore, it necessitates solving Lagrangian Dual (LD) problem to derive a favorable upper bound function given an arbitrary price.

Since it is hard to solve the LD problem parametrically, an implementable policy consisting of the prices set throughout the selling horizon has to be selected. That is, if the length of the selling horizon is *N* periods, the implementable policy is defined as a set of *N* feasible prices. Let the trial decision in period *t* be denoted by $\overline{p_t}$. The criterion to be satisfied in the selection of an implementable policy is the feasibility implying that the decreasing price environment requires the relationship shown below:

$$\overline{p_N} \leq \overline{p_{N-1}} \leq \overline{p_{N-2}} \leq \cdots \leq \overline{p_1}. \quad (2.23)$$

We do not need to derive an upper bound function for the post-decision profit-to-go function of the last period by solving the LD problem because we can derive the closed-form expression. Such a derivation is possible because as explained before, we know the exact closed-form profit function of the dummy period *N+1*. Then, the only upper bound imposed on each iteration over the post-decision profit-to-go function of the last period is:

$$\bar{\varphi}_N(PRIL_N^m, PRIL_N^r, p_N) = \frac{1}{M_N} \cdot \sum_{k=1}^{M_N} \sum_{l=0}^{a_{N,k}} \sum_{m=0}^{a_{N,k}-l} \big(Q_{N+1}\big(PRIL_N^m, PRIL_N^r, p_N, (l,m)\big).$$
$$.\binom{a_{N,k}}{l}.\binom{a_{N,k}-l}{m}.(MS_N^m(p_N))^l.(MS_N^r(p_N))^m.$$
$$.(MS_N^o(p_N))^{a_{t,k}-l-m}. \tag{2.24}$$

Then, we will move on to the penultimate period and derive upper bound functions for the approximate post-decision profit-to-go function for the retailers' all possible inventory levels after replenishments at the beginning of the period. As explained before, given each feasible pair of inventory levels, we have to solve the LD problem of the last period to derive an upper bound function for the pre-decision profit-to-go function of the last period for each possible pair of demands observed by the retailers given a realization generated for the number of potential customers in the market in the penultimate period. Then, we have to repeat the same operation for all the other realizations generated for period *N-1*. When solving the LD problem, we assume that the price set by the manufacturer in period *N-1* is the trial decision $\bar{p}_{N-1}$. The LD problem to be solved to derive an upper bound for the pre-decision profit-to-go function given a feasible pair (I_{N-1}^m, I_{N-1}^r) of inventory levels, a feasible pair (D_{N-1}^m, D_{N-1}^r) of demands observed by the retailers and the trial decision $\bar{p}_{N-1}$ is as follows:

$$\min_{\lambda \geq 0} \; d(\lambda), \tag{2.25}$$

where

$$d(\lambda) = \max_{(p_N,T)\in F_N} \; POD_N(I_{N-1}^r, p_N, D_{N-1}^r) + T + \lambda.(\bar{p}_{N-1} - p_N), \tag{2.26}$$

$$F_N = \{(p_N, T) \in \mathbb{R}^2 \colon T \leq \; \bar{\varphi}_N(IL_N^m(I_{N-1}^m, D_{N-1}^m), IL_N^r(I_{N-1}^r, D_{N-1}^r), p_N), \; p_N \geq 0\}. \tag{2.27}$$

There are some methods proposed to solve LD problem such as subgradient algorithm, Bundle's method, outer linearization etc. Let λ^* be the optimal solution of the LD problem formulated above. The next step is to find an upper bound for an arbitrary price p_{N-1} set by the manufacturer in the previous period. Since λ^* is always a feasible solution, the optimal solution of the Lagrangian relaxation problem shown below provides a part of the upper bound function for an arbitrary price p_{N-1}.

$$d(\lambda^*) = \max_{(p_N,T)\in F_N} POD_N(I^r_{N-1}, p_N, D^r_{N-1}) + T - \lambda^*.p_N, \tag{2.28}$$

where

$$F_N = \{(p_N, T) \in \mathbb{R}^2 : T \le \bar{\varphi}_N(IL^m_N(I^m_{N-1}, D^m_{N-1}), IL^r_N(I^r_{N-1}, D^r_{N-1}), p_N),\ p_N \ge 0\}. \tag{2.29}$$

If we add the functional terms of the price set by the manufacturer in period *N-1* to the optimal solution of the Lagrangian relaxation problem shown above, then it provides an upper bound function of the price (p_{N-1}) set in the penultimate period over the approximate pre-decision profit-to-go function of the last period given the retailers' post-replenishment inventory levels (I^m_{N-1} and I^r_{N-1}) in period *N-1* and the amounts of demand (D^m_{N-1} and D^r_{N-1}) observed by the retailers in period *N-1*. That upper bound can be expressed as follows:

$$\begin{aligned}\bar{Q}_N\left(I^m_{N-1}, I^r_{N-1}, p_{N-1}, (D^m_{N-1}, D^r_{N-1})\right) = {} & PRD_N(I^m_{N-1}, I^r_{N-1}, p_{N-1}, D^m_{N-1}, D^r_{N-1}) \\ & + d(\lambda^*) + \lambda^*.p_{N-1}.\end{aligned} \tag{2.30}$$

We have to repeat the same operations for all possible pairs of demands given each realization generated for the number of potential customers. Then, using Equation (2.21), we can derive the first upper bound function over the approximate post-decision profit-to-go function of period *N-1* for the pair (I^m_{N-1}, I^r_{N-1}) of inventory levels. Likewise, we can derive upper bound functions for all the other feasible pairs of inventory levels.

In the process of finding an upper bound function for the approximate post-decision profit-to-go function of period *N-2* given the inventory levels after replenishments, as explained before, we have to derive upper bound functions for the approximate pre-decision profit-to-go function of period *N-1* given the inventory levels after replenishments in period *N-2* and all possible pairs of demands observed by the retailers in period *N-2* given each realization generated for the number of potential customers in period *N-2.* Given the inventory levels right after the replenishments and the pair of demands observed by the retailers in period *N-2*, we can calculate the inventory levels right after the replenishments in period *N-1.* Then, before solving the LD problem of period *N-1*, we have to impose the upper bound found for the approximate post-decision profit-to-go function of period *N-1* that corresponds to the inventory levels after the replenishments in period *N-1* by adding a constraint to the constraint set. This means that the constraint set of each problem will contain an extra constraint forcing the upper bound over the approximate post-decision profit-to-go function. Let $u_t^k(I_t^m, I_t^r, p_t)$ denote the upper bound function of p_t derived in the *kth* iteration for the approximate post-decision profit-to-go function of period t given the pair (I_t^m, I_t^r) of inventory levels after the replenishments in period *t*. Then, at the end of the backward step of the first iteration, we have to solve the following problem to obtain the first provisional approximate optimal price for the first period and an upper bound over the optimal value of the SAA problem.

$$\max_{(p_1,T)\in F_1} FPP(p_1) + T, \tag{2.31}$$

where

$$F_1 = \{(p_1, T) \in \mathbb{R}^2 : T \leq u_1^1(S_1^m, S_1^r, p_1) \;\; p_1 \geq 0\}. \tag{2.32}$$

Let $\overline{\vartheta_1}$ be the upper bound and (p_1^1, T^*) be the optimal solution attained by solving the problem shown above. If the manufacturer fixes the price at p_1^1, then the expected total profit across all possible demand scenarios bounds the actual optimal value from below. We still do not know whether the optimal solution of the SAA problem we have been solving is p_1^1 or not. This means that pricing the product at p_1^1 in the first period, we can also acquire a lower bound for the optimal value of the SAA problem. In this case, we have to check how close the upper bound $\overline{\vartheta_1}$ and the lower bound are to one another. However, it might be hard to determine the lower bound since the larger number of realizations generated for the number of potential customers in each period at the beginning of the backward step, the larger number of demand scenarios for which to solve the models in a forward fashion. Therefore, we have to commit ourselves to constructing a one-sided confidence interval of the expected total profit made across all the demand scenarios comprising the realizations generated for the SAA problem. The forward step is intended for the construction of the confidence interval.

For the forward step, assume that *M* realizations have been generated for the number of potential customers to simulate the random event of each period. In this case, given that the selling horizon consists of *N* periods, there are a total of M^N scenarios. Since these realizations are sampled from the corresponding Poisson distributions, we know that the probability of observing either scenario among these M^N scenarios is $\frac{1}{M^N}$. A subscenario of a given scenario consists of the feasible distributions of the potential customers to the retailers. Let $\mu_i^j(p_1^1)$ be the profit made if the *j*th subscenario of the *i*th scenario occurs and $P_i^j(p_1^1)$ be the probability of observing the *j*th subscenario of the *i*th scenario given the price p_1^1 set by the manufacturer in the first period of the selling horizon. $P_i^j(p_1^1)$ is exactly the probability of observing the *i*th scenario multiplied by the conditional probability of observing the *j*th subscenario given the *i*th scenario. By definition of the SAA problem, the probability of observing a scenario is $\frac{1}{M^N}$. If the scenario *i* has s_i subscenarios, then the expected total profit across all realizations generated at the beginning of the backward step given the price p_1^1 is provided by:

$$\mu(p_1^1) = \sum_{i=1}^{M^N} \sum_{j=1}^{s_i} P_i^j(p_1^1). \mu_i^j(p_1^1). \tag{2.33}$$

As explained before, Equation (2.33) provides us with a lower bound for the optimal value of the SAA problem. In order to find a lower bound over the lower bound given by Equation (2.33), we uniformly extract K demand subscenarios from a total of $\sum_{i=1}^{M^N} s_i$ subscenarios in the fashion that one of the M^N scenarios is uniformly chosen and then one of the subscenarios of that scenario is extracted. The extraction of the subscenario entails solving the models from the first period to the last one and dealing with the evolution of probabilities. Firstly, we have to calculate the retailers' market shares in the first period given the provisional optimal price p_1^1 and then generate a binomial random variate standing for the demand observed by the manufacturer-controlled retailer in the first period for the corresponding scenario. Then, we have to subtract that random variate from the number of potential customers in the first period given the selected scenario. Then, we have to calculate the probability of a potential customer purchasing the product from the independent retailer given that it does not purchase from the manufacturer-controlled retailer. Then, we have to generate a binomial random variate standing for the demand observed by the independent retailer in the first period. Inserting the generated binomial random variates into the model of the second period, we have to solve it to find the optimal price to be set in that period given the scenario and its subscenario. Afterwards, we have to calculate the retailers' market shares in the second period given the optimal price that has just been attained. Then, we have to generate random variates as we do for the first period. At the end, this process forms a complete subscenario for the previously chosen scenario. Considering the chosen K scenarios, we need a total of K iterations in the forward step. Let $\mu_{[i]}(p_1^1)$

be the total profit made out of the *i*th selected subscenario. Then, $\sum_{i=1}^{K} \frac{1}{K} . \mu_{[i]}(p_1^1)$ is an unbiased estimator of (p_1^1) . Then, the one-sided confidence interval with a confidence level of 1-α is as follows:

$$\sum_{i=1}^{K} \frac{1}{K} . \mu_{[i]}(p_1^1) - z_\alpha . \frac{\sqrt{\left(\sum_{j=1}^{K} \left(\mu_j(p_1^1) - \sum_{k=1}^{K} (1/K) . \mu_{[k]}(p_1^1)\right)^2\right) / K - 1}}{\sqrt{K}} \leq \mu(p_1^1). \quad (2.34)$$

The left-hand side of the inequality (2.34) is also a lower bound for the optimal value of the SAA problem. Let LB_1 denote the lower bound obtained at the end of the forward step of the first iteration. Then, if the inequality (2.35) shown below is satisfied given an acceptably small tolerance ϵ, then p_1^1 is the approximate optimal price to be set in the first period. Otherwise, we have to move on to the second iteration and derive new upper bounds for the post-decision profit-to-go functions using one of the feasible trial solutions obtained in the forward step.

$$\overline{\vartheta_1} - LB_1 < \epsilon. \quad (2.35)$$

If we are currently carrying out the *k*th iteration, we have to derive an extra upper bound for the approximate post-decision profit-to-go function of each period for all possible inventory levels after replenishment. That is, we have to keep all the upper bounds derived in the previous iterations. After implementing the backward step and the forward step, we obtain a new upper bound and a new lower bound for the optimal value of the SAA problem, respectively. If the stopping criterion is satisfied at the end of the iteration k, then p_1^k is the approximate optimal solution of the SAA problem for the first period of the selling horizon. The step-by-step summary of the variant SDDP algorithm is provided in Appendix A.

In the following section, four critically important contractual parameters are evaluated to observe how the changes in each parameter impact the approximate optimal price, the retailers' market shares and their expected total net profits. Some approaches on the selection of the best compromise values of these contractual parameters are also provided. For all these analyses, the variant SDDP algorithm proposed in this section is implemented and the manufacturer is assumed to specify its pricing strategy based on the approximate optimal solutions returned by this algorithm.

2.4 Numerical Experiment

We have to implement the algorithm designed in the previous section for some problem instances to observe the manufacturer's pricing decisions through this approach. There exist some critically pivotal contractual parameters such as reimbursement rate, discount rate, markdown rate and refund per product returned by the independent retailer. Therefore, it is fundamental to analyze the evolution of the manufacturer's pricing strategy over the varying values of these parameters. It is out of scope to determine the optimal values for these contractual parameters but it can be beneficial to get an insight into the selection process for some further research. At this point, the selected values for these contractual parameters have to satisfy both retailers' profit expectations by enabling them to recover their initial outlay on the production or the acquisition of some initial inventory at the beginning of the selling horizon. However, before solving any problem instance, we have to choose a suitable method to estimate the retailers' market shares given the price set by the manufacturer in a given period.

One of the most widely used methods employed to estimate the market shares for existing alternatives in a given market is multinomial logit models (MNL) first proposed by Luce [24] with the derivation of choice probabilities. Then, Luce and Suppes [25] show the connection between the logit choice probability functions and the unobserved utility distributed extreme value. Finally, McFadden [26] finishes off the research by proving that logit choice probability functions always entail the extreme value distribution of the unobserved utility. In this section, we utilize customized forms of the choice probability functions that are built on a price-dependent utility function. As explained before, there are three alternatives a potential customer can select among in our problem setting. Therefore, there exist three choice probability functions associated with a purchase from the manufacturer-controlled retailer, a purchase from the independent retailer and no-purchase option. The probability of a potential customer purchasing the product from the manufacturer-controlled retailer is given by

$$MS_m(p) = \frac{e^{\frac{\mu_m - p}{\tau}}}{e^{\frac{\mu_m - p}{\tau}} + e^{\frac{\mu_r - (1-\gamma).p}{\tau}} + 1}. \tag{2.36}$$

Likewise, the probability of a potential customer purchasing the product from the independent retailer is

$$MS_r(p) = \frac{e^{\frac{\mu_r - (1-\gamma).p}{\tau}}}{e^{\frac{\mu_m - p}{\tau}} + e^{\frac{\mu_r - (1-\gamma).p}{\tau}} + 1}. \tag{2.37}$$

If a potential customer does not prefer purchasing this product from either retailer, then it directly leaves the market since there is no substitute good. The proportion of the mean number of potential customers choosing the no-purchase option to the number of potential customers in the market is provided by

$$MS_o(p) = \frac{1}{e^{\frac{\mu_m - p}{\tau}} + e^{\frac{\mu_r - (1-\gamma).p}{\tau}} + 1}\,. \tag{2.38}$$

As can easily be observed, all the probabilities presented above add up to *1*. In these choice probability functions, μ_m and μ_r stand for the mean maximum price a potential customer is willing to pay to purchase the product from the manufacturer-controlled retailer and the independent retailer, respectively. The maximum-willingness-to-pay values can be elicited from a sample of potential customers by administering a marketing survey to them. In such a survey, some information on delivery times, post-sale services etc. should be provided about the retailers and each potential customer should be asked about the maximum price it is willing to pay to purchase the product from each one of the retailers. Then, the mean maximum price can be estimated for each retailer by the sample average of the maximum-willingness-to-pay values provided by the potential customers.

Another parameter showing up in the choice probability functions is the scale factor τ. In Luce and Suppes [25] and McFadden [26], it is shown that the unobserved utility follows extreme value distribution and the difference between two extreme value random variables is logistically distributed. The variance of a logistic random variable is $\frac{\pi^2.\tau^2}{6}$ so we have to estimate this variance to obtain an estimator of the scale factor. For this purpose, we have to compute the sample variance of the maximum-willingness-to-pay values provided by the potential customers for both the retailers. Then, we can extract the estimator of the scale factor by equating the sample variance with the variance of the logistic random variable.

In this section, we observe the influence of the changes in the value of each critically significant contractual parameter on the approximately optimal price that the manufacturer should set in the first period given the pricing strategy we propose by solving the mathematical models shown in Section 2.3 for a selling horizon of three periods. We also provide some approaches for the selection process of the values of these contractual parameters to render the price protection contract profitable and favorable for both of the retailers. We have done an extreme value analysis for the other parameters than the contractual parameters discussed throughout this section. We have observed that the changes in the value of those parameters have no influence on how the approximately optimal price and the retailers' true expected total profits change in relation to the discussed contractual parameters. The changes in the value of those parameters impact only the amount of change in the approximately optimal price and the retailers' expected total profits. For that reason, we fix those parameters at specific values and do not change them throughout the section. The values that those parameters take on are presented in Table 2.2 shown below.

Table 2.2 The values of fixed contractual and non-contractual parameters

Parameter	Value
Holding cost per dollar per period ($)	0.05
Discount rate for backordered demand (%)	15
Salvage value ($)	60
Production costs ($)	(60, 60, 60)
Manufacturer-controlled retailer's reorder points	(22, 19, 17)
Independent retailer's reorder points	(15, 13, 10)
Mean number of potential customers per period	(22, 19, 15)
Mean maximum-willingness-to-pay values for retailers ($)	(200, 175, 140)
Multinomial logit scale factors	(32.66, 27.45, 25.55)
Allowable amounts of backordered demand for retailers	(15, 15, 0)

Apart from contractual and non-contractual parameters, there exist some parameters the values of which we have to fix to implement the variant SDDP algorithm. We generate 15 Poisson random variates standing for the number of potential customers in the market for each period of the selling horizon. We form 100 demand subscenarios in the forward step as explained in the previous section to obtain a lower bound for the optimal value of the actual SAA problem. We also have to define the stopping criterion to check whether to stop implementing the algorithm at the end of each iteration or not. As the stopping criterion, we would like the upper bound obtained at the end of the backward step to be in 10% neighborhood of the absolute value of the lower bound obtained at the end of the backward step.

The first contractual parameter we examine is the reimbursement rate. As defined before, reimbursement rate is the proportion of the independent retailer's on-hand inventory that is eligible for reimbursement in case of a reduction in the price set by the manufacturer in a given period. Although it seems to take on values ranging between 0% and 100%, it might also take on a value strictly larger than 100% under some special circumstances. Therefore, we solve problem instances for various values of the reimbursement rate by setting discount rate to 40%, markdown rate to 20% and refund per returned product to 200$. The influence of the changes in the reimbursement rate on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 2.3 for reimbursement rates not larger than 100% and in Table 2.4 for reimbursement rates above 100%.

Table 2.3 The change of approximate optimal price and market shares in relation to the reimbursement rate up to 100%

Reimbursement rate (%)	40	60	70	80	100
Approximate optimal price ($)	176.37	176.22	176.14	176.06	175.89
Proportion of lost customers (%)	10.95	10.91	10.89	10.87	10.83
Manufacturer's market share (%)	22.57	22.60	22.61	22.63	22.65
Independent retailer's market share (%)	66.48	66.49	66.50	66.50	66.52

Table 2.4 The change of the approximate optimal price and market shares in relation to the reimbursement rate above 100%

Reimbursement rate (%)	150	400	550	700	850	1300
Approximate optimal price ($)	175.44	172.14	169.36	166.38	163.75	154.93
Proportion of lost customers (%)	10.71	9.92	9.29	8.66	8.13	6.56
Manufacturer's market share (%)	22.73	23.28	23.74	24.23	24.66	26.08
Independent retailer's market share (%)	66.56	66.80	66.97	67.11	67.21	67.36

We can easily deduce from Table 2.3 and Table 2.4 that as the reimbursement rate increases, the manufacturer tends to reduce the price. The first reason for such a tendency is that the manufacturer avoids possibly higher amounts of reimbursement in the following periods of the selling horizon. The manufacturer raises the independent retailer's market share by diminishing the price so as to boost demand the independent retailer observes because higher amount of demand means lower amount of on-hand inventory held by the independent retailer at the end of the period. Another reason is that as the price decreases, the manufacturer-controlled retailer's market share also increases. In this case, the demand observed by the manufacturer-controlled retailer is on the upswing, which attenuates the negative impact stemming from the decreasing selling price.

Since the product in question is a slow-moving *A* item, the reorder points are relatively low. In this case, reimbursement rates below 100% might be unsatisfactory for the independent retailer because of a possibly low amount of remaining on-hand inventory it holds at the end of each period. Therefore, the manufacturer and the independent retailer might negotiate reimbursement rates above 100% at the beginning of the selling horizon to make the price protection contract more appealing for the independent retailer. However, both parties have to be pleased with the expected total profit they make throughout the selling horizon. For this reason, we also have to observe the effect of the changes in the reimbursement rate on the expected total profits. For this purpose, we form 100 demand subscenarios by generating demand realizations from the true distributions in the forward step just as we do to obtain a lower bound for the optimal value of the SAA problem. However, the way how the demand realizations are generated is different in this case.

Given the price set by the manufacturer in the first period, we can easily calculate the retailers' market shares employing the aforementioned Equation (2.36) and Equation (2.37). Since the number of potential customers in a given period is Poisson distributed, the demand observed by each retailer is also Poisson distributed. Then, we can determine the mean demand observed by a given retailer multiplying the known mean number of potential customers in the first period by its market share derived from the corresponding choice probability function. This means that when sequentially solving the mathematical models, we have to generate a Poisson random variate representing the demand observed by each retailer given the price set in the given period. In this way, we draw a sample of 100 profit values for each retailer. Then, we construct one-sided and two-sided confidence intervals of the expected total profit made by each retailer with a confidence level of 80% to observe the impact of various reimbursement rates.

As can be observed in Figure 2.1, as the reimbursement rate rises, the bounds over the independent retailers' expected total net profit increase until the reimbursement rate reaches a specific value and then they start decreasing. This means that extremely high reimbursement rates are less favorable than moderate reimbursement rates. If it is more highly weighted how much profit the independent retailer makes throughout the selling horizon, then the best choice turns out to be 400% among all the tried alternatives. The reimbursement rates of 150% and 550% return almost the same results and they are not dominated by the reimbursement rate of 400%. This means that they are also good choices for the independent retailer's profitability.

The evolution of the bounds over the manufacturer's expected total net profit is shown in Figure 2.1. The confidence intervals of the manufacturer's expected total profit are mostly overlapping until the reimbursement rate reaches 70%. We do not observe any dramatic shift of the manufacturer's expected total profit as the value taken on by the reimbursement rate ranges between 100% and 850%. We can state that the changes in the value of reimbursement rate do not have an enormous influence on the manufacturer's expected total profit. Therefore, it is less critical to reckon with the manufacturer's minimum allowable expected total profit than the independent retailer's minimum allowable expected total profit when selecting a compromise value for the reimbursement rate. If it is more highly weighted how much profit the manufacturer-controlled retailer makes throughout the selling horizon, then the best choice turns out to be 60% among all the tried alternatives. The reimbursement rates of 40%, 70% and 150% are also some good options for the manufacturer's profitability. If both parties lean towards relinquishing some revenue, then the reimbursement rates of 150% and 70% can be good options as a compromise solution. Of course, one of the most important things is to settle on the minimum allowable expected profit for each retailer because it also has an influence on the best compromise solution.

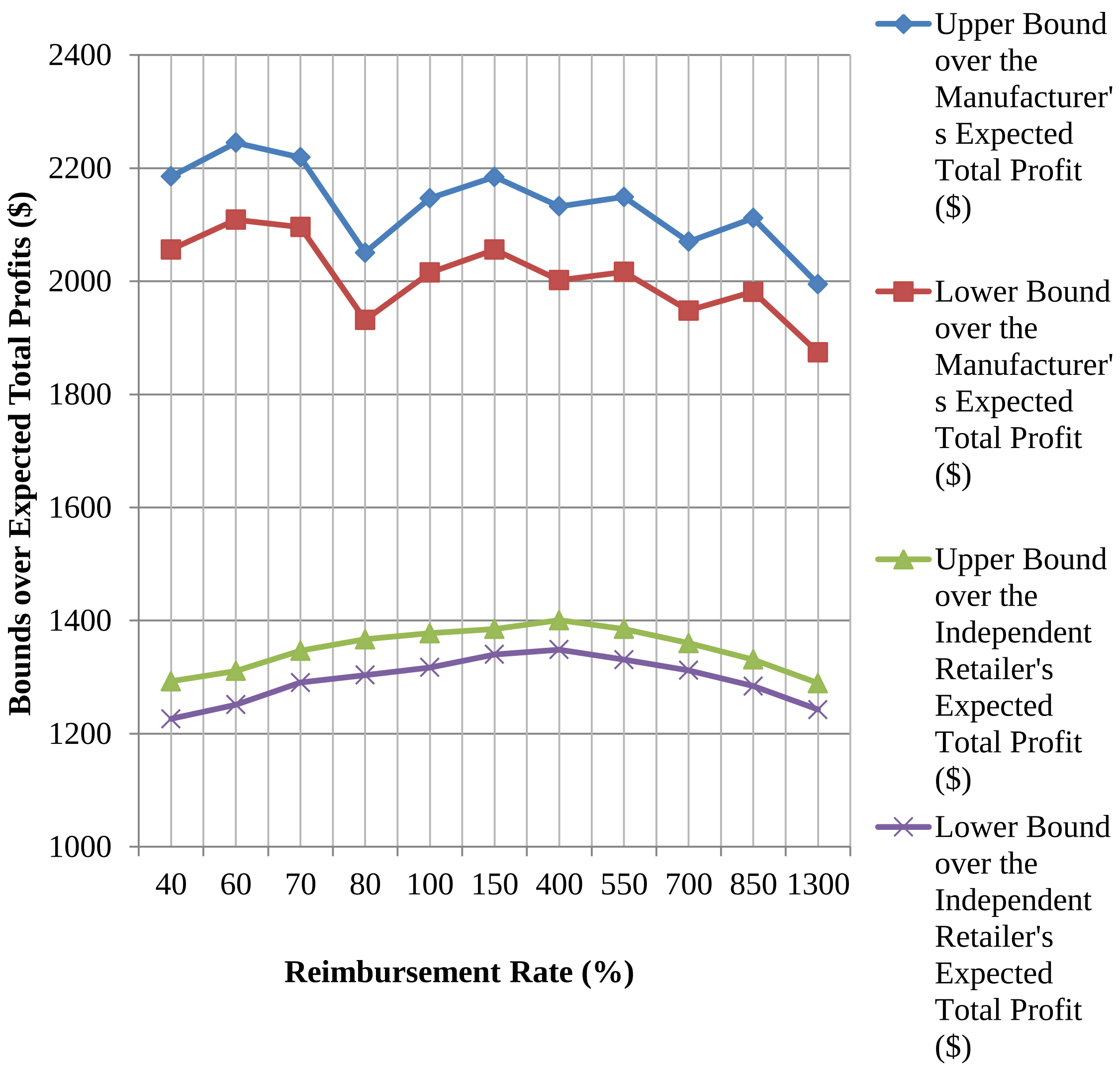

Figure 2.1 The evolution of the bounds over the retailers' expected total net profits in relation to reimbursement rate

Another significant contractual parameter to be assessed is markdown rate. We solve problem instances for various feasible values of markdown rate by setting discount rate to 60%, reimbursement rate to 70% and refund per returned product to

200$. The influence of the changes in the markdown rate on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 2.5.

Table 2.5 The change of the approximate optimal price and market shares in relation to markdown rate

Markdown rate (%)	0	10	20	30	40	50
Approximate optimal price ($)	158.31	160.49	163.53	169.39	183.52	216.84
Proportion of lost customers (%)	12.24	10.17	8.08	6.39	5.45	5.32
Manufacturer's market share (%)	43.88	34.10	24.69	16.31	9.03	3.50
Independent retailer's market share (%)	43.88	55.73	67.23	77.30	85.52	91.18

We can infer from Table 2.5 shown above that as the markdown rate rises, the manufacturer proposes to increase the price. Since higher markdown rates mean higher market shares for the independent retailer and the manufacturer controlled retailer's market share is negatively correlated, the manufacturer's goal is to counteract the financial loss arising from the decreasing market share by increasing the price. At the same time, since the independent retailer's market share rises, the amount of the remaining on-hand inventory the independent retailer holds at the end of the period is expected to be relatively low. Therefore, the manufacturer also tends to reduce the reimbursement cost. In order to observe the impact of markdown rate on the retailers' expected total net profits, we draw a sample of 100 profit values for each retailer in the same way as we do for the reimbursement rate. Then, we construct one-sided and two-sided confidence intervals of the expected total profit made by each retailer with a confidence level of 80%.

As can be seen in Figure 2.2, as markdown rate increases, the confidence interval of the manufacturer's expected total net profit shifts down until the markdown rate reaches a specific value and then it stars shifting up. The reason for such a trend is that the increase in the price and the amounts of replenishment orders placed by the independent retailer counterbalance the negative impact of the decrease in the manufacturer's market share at a specific value of the markdown rate.

As markdown rate increases, the confidence interval of the independent retailer's expected total net profit shifts down as shown in Figure 2.2 because the price at which the independent retailer sells the product to the end customer decreases although its market share increases. It seems to be the best option that both retailers sell the product at the same price by setting the markdown rate to 0% since it provides the highest profit for the manufacturer and the possibly highest profit for the independent retailer. The markdown rate of 10% is almost as profitable for the independent retailer as the markdown rate of 0%. However, the increase of the markdown rate from 0% to 10% drastically cuts down the manufacturer's expected total net profit. We can also deduce that the independent retailer makes larger profit than the manufacturer does until the markdown rate reaches 50%. Therefore, if the profit made by the manufacturer is more highly weighted, there are no many options to choose from.

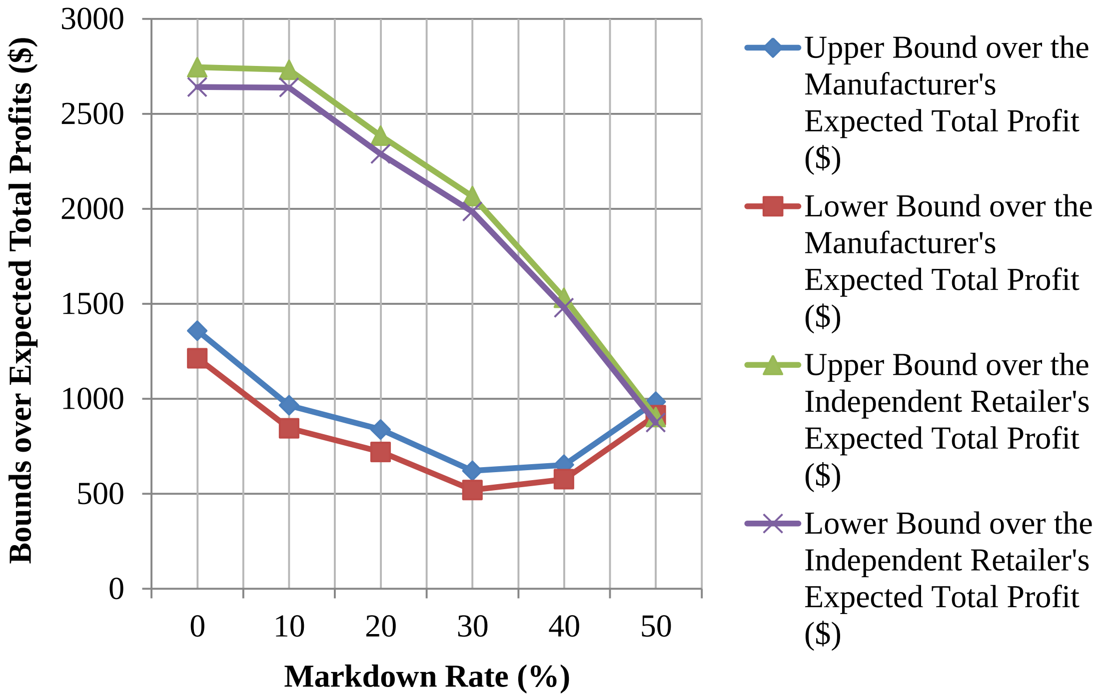

Figure 2.2 The evolution of the bounds over the retailers' expected total net profits in relation to markdown rate

If the discount rate is fixed at 40%, then there are more options in which the manufacturer makes larger profit than the independent retailer does. In case the discount rate is set to 40%, the bounds of the one-sided and two-sided confidence intervals of the manufacturer's and the independent retailer's expected total net profits are as shown in Table 2.6. As can be seen in the table, the markdown rates of 0% and 10% dominate the markdown rates of 20% and 30% since the increase in the markdown rate from 10% weighs down both retailers' profitability. If a profit of around 1610$ is sufficient for the independent retailer, then the markdown rate of 10% can be chosen since at that value, the manufacturer seems to make the possibly highest profit. Otherwise, a markdown rate between 0% and 10% has to be selected for the independent retailer to make higher profit but in that case, the manufacturer has to renounce some of its revenue. This means that the minimum allowable expected total net profits and the profit made by which retailer is more highly weighted are very critical evaluation measures in the selection process of the markdown rate to determine the best compromise solution.

Table 2.6 The influence of the markdown rate on the bounds of the confidence intervals of the manufacturer's and the independent retailer's expected total net profits in case of the discount rate of 40%

Markdown rate (%)	0	10	20	30
Upper bound of two-sided CI of manufacturer's expected total profit ($)	2133.8	2300.8	2218.9	2217.1
Lower bound of two-sided CI of manufacturer's expected total profit ($)	1978.5	2154	2095.9	2090.3
Lower bound of one-sided CI of manufacturer's expected total profit ($)	2005.2	2300.8	2117	2112.4
Upper bound of two-sided CI of independent retailer's expected total profit ($)	1948.4	1674.3	1346.7	780.4
Lower bound of two-sided CI of independent retailer's expected total profit ($)	1880.9	1610.9	1290.5	733.1
Lower bound of one-sided CI of independent retailer's expected total profit ($)	1892.5	1621.8	1300.2	741.2

The third contractual parameter of which the retailers have to compromise on the value is discount rate. Just as in the analyses of the preceding two contractual parameters, we solve problem instances for various feasible values of discount rate by setting markdown rate to 10%, reimbursement rate to 70% and refund per returned product to 200$. The influence of the changes in the discount rate on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 2.7.

Table 2.7 The change of the approximate optimal price and market shares in relation to discount rate

Discount rate (%)	20	30	40	50	60	70
Approximate optimal price ($)	186.03	176.59	170.12	165.30	160.49	155.17
Proportion of lost customers (%)	19.07	15.23	12.98	11.51	10.17	8.86
Manufacturer's market share (%)	29.24	31.20	32.43	33.28	34.10	34.94
Independent retailer's market share (%)	51.69	53.57	54.59	55.21	55.73	56.20

As we can observe from Table 2.7, the manufacturer lessens the price as the discount rate goes up. The manufacturer increases its market share by taking this action, thereby observing higher demand. Likewise, the independent retailer's market share also increases as the discount rate increases implying that the replenishment orders placed by the independent retailer are larger in this case. Larger replenishment orders mean the independent retailer's less remaining on-hand inventory at the end of each period so the manufacturer reduces the reimbursement cost it incurs, as well. Therefore, the increase in the demand observed by the manufacturer and the increase in the amounts of replenishment orders placed by the independent retailer partly mitigates the negative impact of the decrease in the selling price and the discounted price offered to the independent retailer.

As discount rate increases, the confidence interval of the manufacturer's expected total net profit shifts down as can be observed in Figure 2.3. On the contrary, the confidence interval of the independent retailer's expected total net profit shifts up because although the marked down price at which it sells the product to the end customer decreases, the downturn in the discounted price offered by the manufacturer for replenishment orders is more precipitous. Therefore, the independent retailer's profit per product increases in this case. Furthermore, the rise in the discount rate also triggers a rise in the independent retailer's market share, as well and the opposite impact of the decreasing reimbursement revenue does not hurt the independent retailer too much.

The manufacturer's expected total net profit is negatively correlated with the independent retailer's expected total net profit as can be inferred from Figure 2.3. This means that there exists no dominated solution and there is a trade-off between the options. As the independent retailer's expected total net profit is higher than the manufacturer's expected total net profit at the discount rate of 50%, the manufacturer's expected total net profit surpasses when the discount rate is set to 40%. This means that there exists a balance point between these values. If the manufacturer's expected total net profit is more highly weighted, then the discount rate has to be below this balance point. On the contrary, if the independent retailer's expected total net profit is more highly weighted, then the discount rate has to be above the balance point. However, in the selection of the best compromise solution, the minimum allowable profits are also significant evaluation measures as explained before.

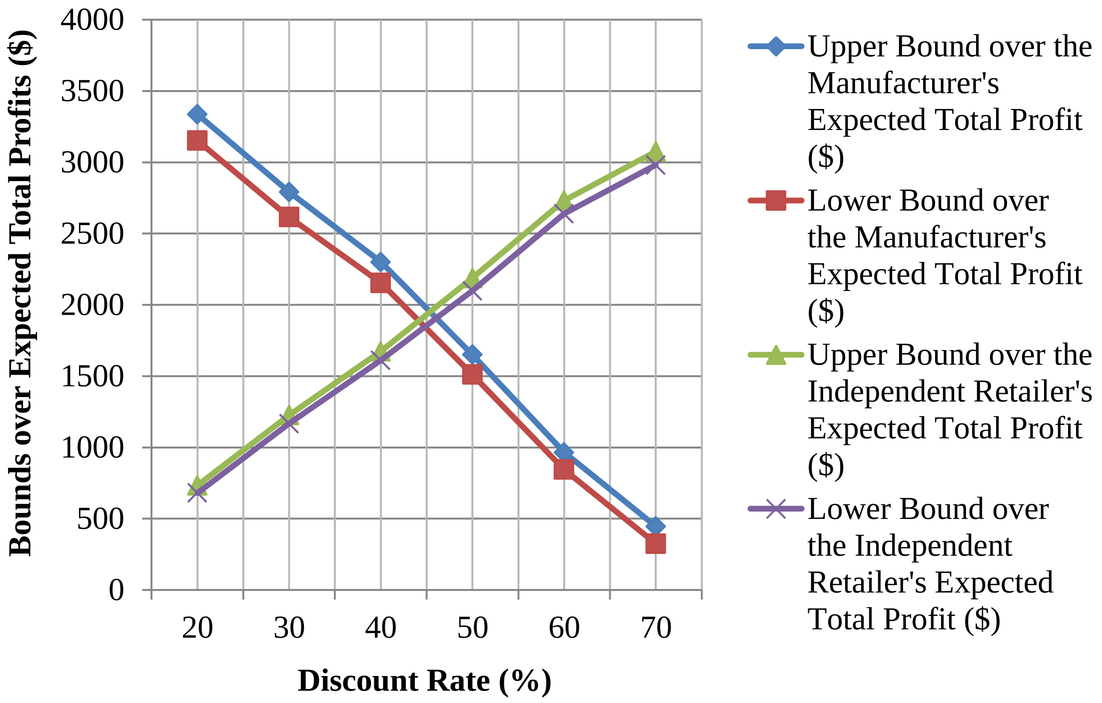

Figure 2.3 The evolution of the bounds over the retailers' expected total net profits in relation to discount rate

The last critical contractual parameter to be analyzed is the refund per product returned by the independent retailer at the end of the selling horizon. Just as in the analyses of the previous three contractual parameters, we solve problem instances for some values of refund by setting markdown rate to 80%, reimbursement rate to 70% and discount rate to 60%. The influence of the changes in the refund on the approximately optimal price the manufacturer sets, the retailers' market shares and the proportion of the lost customers in the first period is shown in Table 2.8.

Table 2.8 The change of the approximate optimal price and market shares in relation to refund per returned product at the end of the selling horizon

Refund ($)	0	50	100	150	200	250
Approximate optimal price ($)	185.0037	185.4432	176.1405	176.1416	176.1427	176.1437
Proportion of lost customers (%)	13.3392	13.4718	10.8913	10.8916	10.8919	10.8922
Manufacturer's market share (%)	21.1127	21.0375	22.6126	22.6124	22.6123	22.6121
Independent retailer's market share (%)	65.5481	65.4907	66.4961	66.4960	66.4958	66.4957

As we can observe from Table 2.8, the manufacturer increases the price as the refund per product rises until it reaches a specific value. That value seems to be between 50 and 100 in this case. Then, the approximate optimal price shows a decreasing pattern until the refund reaches another breaking point. That breaking point is above the first breaking point and less than 100 in this case. After the refund surpasses the second breaking point, the approximate optimal price seems to increase very slightly as the refund rises. After the second breaking point, the manufacturer tries to increase the revenue generated by selling the product to the end customer at higher prices in purpose for covering the increase in the refund per product. However, since the independent retailer's market share decreases as the price set by the manufacturer increases, the independent retailer is likely to be possessed of higher amount of remaining on-hand inventory at the end of the selling horizon. This might lead to a rise in total refund. For that reason, the manufacturer is conservative in pricing and avoids dramatic increases after the second breaking point.

After the second breaking point, the market shares fluctuate and the deviations are unnoticeably small. Since the low values of refund do not weigh on the manufacturer's profitability too much, the manufacturer tends to increase the price more steeply compared to the higher values of refund although it decreases the manufacturer's and the independent retailer's market shares.

As refund increases, the confidence interval of the manufacturer's expected total net profit shifts down as can be observed in Figure 2.4 since the total refund increases and the manufacturer's actions can partly compensate for that increase. Obviously, the most preferable value of refund is 0$ among the evaluated alternatives for the manufacturer but the independent retailer is possibly dissatisfied with this value since its expected total net profit is low in that case. On the contrary, the confidence interval of the independent retailer's expected total net profit shifts up. Apparently, the best choice is 250$ among the evaluated alternatives for the independent retailer but the manufacturer-controlled retailer might be dissatisfied this time since it is required to forgo some profit in that case. We can easily deduce that the balance point is not reached yet and refund can be increased more until the independent retailer's expected total net profit becomes level with the manufacturer's expected total net profit. If the manufacturer's profit is more highly weighted, then the refund has to be less than that balance point. Otherwise, the refund has to be more than or equal to the balance point. Of course, the minimum allowable profits are critical evaluation measures in search for the best compromise solution.

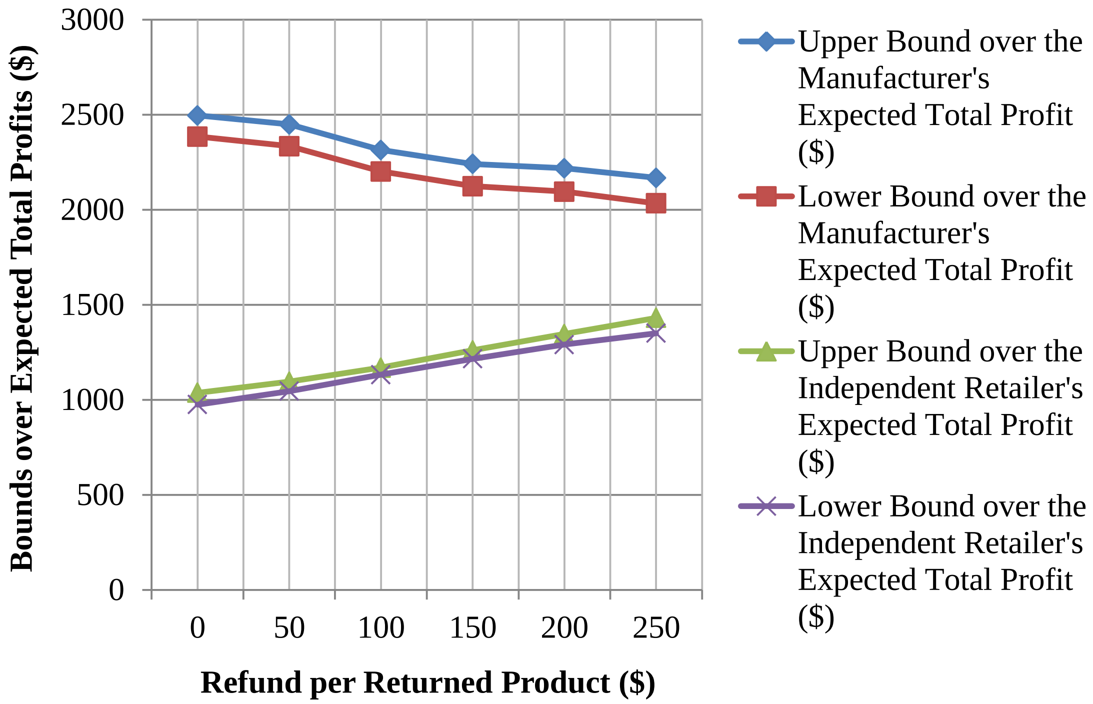

Figure 2.4 The evolution of the bounds over the retailers' expected total net profits in relation to refund per returned product

The findings on the impact of an increase in the value of each critical contractual parameter on approximately optimal price and the retailers' true expected total net profits are summarized in Table 2.9 shown below.

Table 2.9 Impact of an increase in the values of the critical contractual parameters on approximately optimal price and the retailers' true expected total net profits

	Approximately optimal price	Confidence interval of manufacturer's expected total net profit	Confidence interval of independent retailer's expected total net profit
Reimbursement rate	Slightly decreasing	Alternating	First shifting up, then shifting down
Markdown rate	Increasing	Following trajectory or inverted trajectory	Shifting down
Discount rate	Decreasing	Shifting down	Shifting up
Refund	First alternating, then slightly increasing	Shifting down	Shifting up

In conclusion, the algorithm takes an acceptably small number of iterations to converge. The maximum observed number of iterations in the data collection process is six and it has occurred only once. The running time of a single iteration of the algorithm has been around one and half an hour and we have observed reasonable amount of deviations when different combinations of values have been assigned to the problem parameters. The running time dwindles from iteration to iteration because of the extra upper bound constraints added to each constraint set in the backward step of the algorithm. Compared to the approximate dynamic programming algorithms proposed to get around three curses of dimensionality, the running time per iteration is longer since the algorithm traverses all the possible pairs of inventory levels with which the retailers can start a given period instead of visiting a single state on each iteration. However, it deals with the estimation of the post-decision profit-to-go functions of a given period in a more skillful way by deriving upper bound functions of retail price set in the previous period instead of visiting a single retail price on each iteration. Considering that approximate dynamic programming algorithms are bound to necessitate an undue number of iterations for a decent approximation, a longer running time per iteration of the variant SDDP algorithm is tolerable.

By the inferences from the analyses done in this section, the selection of a compromise value for the discount rate and the markdown rate is essential to ensure high profitability for both the manufacturer and the independent retailer given the approximately optimal pricing strategy proposed in this study. The changes in the values of reimbursement rate and refund per returned product do not have a massive impact on the manufacturer's expected total profit since the manufacturer can keep a tight grip on the market shares by updating its pricing strategy. However, the selection of ideal values for these two contractual parameters is significant to provide the independent retailer with high enough profitability so that it is convinced to keep the inventory of the product. The most critical thing is to avoid inordinately high values of reimbursement rate since the independent retailer sustains financial loss in that case. The retailers have to determine their minimum allowable profit values that they will stipulate in the contract negotiations because those values are very critical in the selection of the best compromise values of the contractual parameters. In the selection process, trade-offs have to be reckoned with scrupulously, as well. Furthermore, the weights assigned to the retailers' expected total net profits have a conspicuous impact on the best compromise values. Therefore, whether the price protection contract is as profitable for both parties as expected or not depends on the retailers' profit expectations implying that the suitable selection of the contractual parameter values and the accuracy in the requirements of the price protection contract are very decisive.

Chapter 3

Periodic-review Approximately Optimal Pricing in the Presence of Price Protection, End-of-life Return Opportunity and Mid-life Return Opportunities

In this chapter, firstly, the differences in the problem definition compared to the problem discussed in Chapter 2 are provided by defining the boundaries of the research built on some assumptions. Secondly, the stochastic programming model to be solved to determine the manufacturer's optimal pricing strategy is presented and it is discussed whether any changes are needed in the implementation of the variant SDDP algorithm proposed in Chapter 2. Finally, the results of the numerical experiments carried out to observe how the changes in some contractual parameters playing a significant role in the manufacturer's pricing decisions impact the approximate optimal price, the retailers' market shares and their true expected total net profits are presented.

3.1 Problem Definition

In this section, we include mid-life returns in the price commitment policy discussed in the previous section as a supplementary privilege offered to the independent retailer by the manufacturer. We assume that the manufacturer-controlled retailer manages its stock through order-up-to inventory replenishment policy and the independent retailer adopts a hybrid policy that is a combination of order-up-to and dispose-down-to replenishment policies. This assumption is inspired by the results of some research papers in the literature. In Lee et. al. [1], Lee and Rhee [7] and Liu et. al. [22], the order-up-to inventory policy is proven to be optimal in case a retailer is allowed two buying opportunities in a selling horizon of two periods as Chen and Xiao [6] show that dispose-down-to policy is optimal if a retailer is allowed to return some inventory between periods in a two-period case.

If the independent retailer decides to lower its inventory level to the dispose-down-to level at the beginning of any period, it returns products to the manufacturer and gets refunded. In that case, the manufacturer reimburses the independent retailer for the remaining on-hand stock after returning the products if it reduces the retail price. If the independent retailer returns some products and the manufacturer-controlled retailer places a replenishment order at the beginning of a period, the manufacturer satisfies the entire order or a part of it from those returned products depending on whether returned products cover the entire order or not. The excess inventory is not kept but salvaged immediately.

3.2 Model and Methodology

The models constructed for the price commitment policy discussed in Chapter 2 has to be adapted to the new price commitment policy. In this section, we present the new models and discuss the slight change in the implementation of the SDDP algorithm for the numerical experiment. Table 3.1 provided below presents the notation in this section.

Table 3.1 Notation

$\boldsymbol{R}$	Inventory holding cost per dollar per period
$\boldsymbol{\theta}$	Discount rate per product backordered
$\boldsymbol{\beta}$	Discount rate per product ordered by the independent retailer
$\boldsymbol{\alpha}$	Reimbursement rate
$\boldsymbol{c_t}$	Production cost per product in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t^j}$	Demand observed by retailer j in period t $\forall t \in \{1,2,\dots N\}, \forall j \in \{m,r\}$
$\boldsymbol{D_t}$	Ordered pair of demands observed by retailers in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{N_t^j}$	Allowable number of products backordered by retailer j in period t $\forall t \in \{1,2,\dots N\}, \forall j \in \{m,r\}$
$\boldsymbol{S_t^m}$	Manufacturer-controlled retailer's order-up-to level in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{S_t^r}$	Independent retailer's dispose-down-to level in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{s_t^r}$	Independent retailer's order-up-to level in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{w_t^m}$	The value per product salvaged by the manufacturer in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{w_t^r}$	Refund per product returned by the independent retailer in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{I_t^j}$	Retailer j's post-replenishments inventory level in period t $\forall t \in \{1,2,\dots N\}, \forall j \in \{m,r\}$
$\boldsymbol{p_t}$	Retail price set by the manufacturer in period t $\forall t \in \{1,2,\dots N\}$

The manufacturer-controlled retailer raises its inventory level to the order-up-to level (S_1^m) at the beginning of the first period. The independent retailer's order-up-to level (s_1^r) and dispose-down-to level (S_1^r) are assumed to be equal in the first period since the independent retailer has no inventory of the product. Just as the manufacturer-controlled retailer does, the independent retailer raises its inventory level to the order-up-to level (s_1^r) by placing a replenishment order. The independent retailer pays the manufacturer the wholesale price per product which is the discounted retail price. Both retailers observe some demand after the pricing and inventory decisions in the first period. Therefore, the objective function consists of the manufacturer's post-replenishment profit function of the retail price (p_1) and the post-decision profit-to-go function ($\varphi_1(S_1^m, s_1^r, p_1)$) that returns the expected total profit that the manufacturer makes after the replenishments in the first period till the end of the selling horizon given the retailers' post-replenishment inventory levels and the retail price in the first period. The post-decision profit-to-go function ($\varphi_1(S_1^m, s_1^r, p_1)$) of the first period is equivalent to the expectation of the pre-decision profit-to-go function ($Q_2(S_1^m, s_1^r, p_1, D_1)$) of the second period over the ordered pair (D_1) of demands observed by the retailers. The pre-decision profit-to-go function of the second period returns the expected total profit that the manufacturer makes after the replenishments in the first period till the end of the selling horizon given the retailers' post-replenishment inventory levels ($S_1^m$and s_1^r), the retail price (p_1) and the ordered pair (D_1) of demands observed by the retailers in the first period. The model meant to determine the optimal retail price that the manufacturer should set in the first period to maximize its expected total profit in the selling horizon is as presented below:

$$\max_{p_1 \in A_1} -S_1^m . c_1 + \big((1-\beta). p_1 - c_1\big). s_1^r + \varphi_1(S_1^m, s_1^r, p_1), \tag{3.1}$$

where

$$A_1 = \{p_1 \in \mathbb{R} : p_1 \geq 0\}, \tag{3.2}$$

$$\varphi_1(S_1^m, s_1^r, p_1) = \mathbb{E}_{D_1}[Q_2(S_1^m, s_1^r, p_1, D_1)]. \tag{3.3}$$

The pre-decision profit-to-go function ($Q_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1})$) of a given intermediate period *t* that returns the manufacturer's expected total profit following the retailers' replenishment and return decisions in the previous period *t-1* till the end of the selling horizon given the retailers' post-replenishment or post-return inventory levels ($I_{t-1}^m$and I_{t-1}^r), the retail price (p_{t-1}) and the ordered pair (D_{t-1}) of demands observed by the retailers in period *t-1* is equivalent to the optimal value of the following model:

$$Q_t(I^m_{t-1}, I^r_{t-1}, p_{t-1}, D_{t-1}) = \max_{p_t \in A_t} -max\{min\{D^m_{t-1} - I^m_{t-1}, N^m_{t-1}\}, 0\}.\theta.p_{t-1} +$$
$$+ min\{D^m_{t-1}, I^m_{t-1} + N^m_{t-1}\}.p_{t-1} -$$
$$- max\{I^m_{t-1} - D^m_{t-1}, 0\}.c_{t-1}.r -$$
$$- max\{S^m_t - I^m_{t-1} + D^m_{t-1}, 0\}.c_t +$$
$$+ max\{D^m_{t-1} - I^m_{t-1} - N^m_{t-1}, 0\}.c_t +$$
$$+ min\big\{max\{S^m_t - I^m_{t-1} + D^m_{t-1}, 0\} - max\{D^m_{t-1} - I^m_{t-1} - N^m_{t-1}, 0\}, \max\{I^r_{t-1} - D^r_{t-1} - s^r_t, 0\}\big\}.c_t -$$
$$- min\{max\{S^m_t - I^m_{t-1} + D^m_{t-1}, 0\} - max\{D^m_{t-1} - I^m_{t-1} - N^m_{t-1}, 0\}, \max\{I^r_{t-1} - D^r_{t-1} - s^r_t, 0\}\}.w^m_t -$$
$$- \max\{I^r_{t-1} - D^r_{t-1} - s^r_t, 0\}.(w^m_t - w^r_t) +$$
$$+ max\{S^r_t - I^r_{t-1} + D^r_{t-1}, 0\}.\big((1-\beta).p_t - c_t\big) -$$
$$- max\{D^r_{t-1} - I^r_{t-1} - N^r_{t-1}, 0\}.\big((1-\beta).p_t - c_t\big) -$$
$$- max\{min\{I^r_{t-1} - D^r_{t-1}, s^r_t\}, 0\}.\alpha.(p_{t-1} - p_t) +$$
$$+ \varphi_t(IL^m_t(I^m_{t-1}, D^m_{t-1}), IL^r_t(I^r_{t-1}, D^r_{t-1}), p_t), \quad (3.4)$$

where

$$A_t = \{p_t \in \mathbb{R}: p_t \le p_{t-1}, p_t \ge 0\}, \quad (3.5)$$

$$\varphi_t(IL^m_t(I^m_{t-1}, D^m_{t-1}), IL^r_t(I^r_{t-1}, D^r_{t-1}), p_t) = \mathbb{E}_{D_t}[Q_{t+1}(IL^m_t(I^m_{t-1}, D^m_{t-1}), IL^r_t(I^r_{t-1}, D^r_{t-1}), p_t, D_t)], \quad (3.6)$$

$$IL^m_t(I^m_{t-1}, D^m_{t-1}) = max\{S^m_t, I^m_{t-1} - D^m_{t-1}\}, \quad (3.7)$$

$$IL^r_t(I^r_{t-1}, D^r_{t-1}) = \begin{cases} s^r_t & if\ I^r_{t-1} - D^r_{t-1} < s^r_t, \\ I^r_{t-1} - D^r_{t-1} & if\ s^r_t \le I^r_{t-1} - D^r_{t-1} \le S^r_t, \\ S^r_t & if\ I^r_{t-1} - D^r_{t-1} > S^r_t. \end{cases} \quad (3.8)$$

The objective function of the model shown above contains the profit that the manufacturer makes by selling some products to the end customer in period *t-1* and meeting the demand backordered in the period *t-1*. It also includes a profit function of the retail price (p_t) set in period t that returns the profit the manufacturer makes by selling products to the independent retailer at the beginning of the period t in case of a replenishment order. The inventory holding cost incurred because of the inventory carried over to period t, the cost that springs from refunding the independent retailer the products returned by the independent retailer to reduce its inventory level to the dispose-down-to level, the production cost, the cost of reimbursement required by price protection policy, the salvage value of the excess inventory that the manufacturer has after the delivery of the returned products to meet the manufacturer-controlled retailer's replenishment order and the post-decision profit-to-go function of period t are the other elements of the objective function. In case of a replenishment order from the manufacturer-controlled retailer, whether or not the products returned by the independent retailer are sufficient to meet the manufacturer-controlled retailer's replenishment order is also dealt with.

We can directly calculate the profit that the manufacturer makes after the pricing and inventory decisions in the last period given the retailers' post-replenishment or post-return inventory levels (I_N^m and I_N^r), the retail price (p_N) set by the manufacturer and the ordered pair (D_N) of demands observed by the retailers in the last period by evaluating the function shown below:

$$\begin{aligned} Q_{N+1}(I_N^m, I_N^r, p_N, D_N) \ = \ & -max\{I_N^m - D_N^m, 0\}. c_N. r + D_N^m. p_N - \ max\{D_N^m - I_N^m, 0\}. p_N \\ & + max\{I_N^m - D_N^m, 0\}. w_{N+1}^m \\ & + max\{I_N^r - D_N^r, 0\}. (w_{N+1}^m - w_{N+1}^r). \end{aligned} \tag{3.9}$$

The expectation of the function shown above over the ordered pair (D_N) of demands observed by the retailers is exactly the post-decision profit-to-go function ($\varphi_N(I_N^m, I_N^r, p_N)$) of the last period.

There is a small difference in the implementation of the SDDP algorithm for this price commitment policy. In the backward step, we derive an upper bound over the post-decision profit-to-go function of each period for each possible pair of inventory levels with which the retailers can start the corresponding period as explained in the previous report. In this case, the inventory level with which the independent retailer can start a given period ranges between the order-up-to level and dispose-down-to level.

3.3 Numerical Experiment

In this section, the variant SDDP algorithm is employed to estimate and assess the impact of some contractual parameters on the approximately optimal pricing strategy and the retailers' expected total true profits in a selling horizon of three periods. As explained before, the objectives of this research paper do not involve the optimization of contractual parameters. However, we make evaluations on the selection of ideal values among some alternative ones for the contractual parameters to ensure the profitability of the price commitment policy for both retailers. For this analysis, we employ the same market share model as in the numerical experiment of the previous price commitment policy to reckon with the influence of the retail prices on the mean demand observed by each retailer in an efficient way.

Throughout the section, we observe how the changes in some contractual parameters affect the retail price that the manufacturer sets for the first period of the selling horizon and interpret the confidence intervals of true expected total profits that the retailers make to show how the ideal values can be determined through a comparison technique. For this purpose, we implement the variant SDDP algorithm for different values of each parameter we examine and keep the other parameters at their pre-set values throughout the section. The values that those parameters take on unless otherwise stated are presented in Table 3.2 shown below.

Table 3.2 The values of fixed contractual and non-contractual parameters

Parameter	Value
Holding cost per dollar per period ($)	0.05
Discount rate for backordered demand (%)	15
Salvage values at the end of periods ($)	(60, 60, 60)
Production costs ($)	(60, 60, 60)
Manufacturer-controlled retailer's order-up-to levels	(22, 19, 17)
Independent retailer's order-up-to levels	(40, 20, 9)
Independent retailer's dispose-down-to levels	(40, 26, 12)
Mean number of potential customers per period	(22, 19, 15)
Mean maximum-willingness-to-pay values for retailers ($)	(200, 175, 140)
Multinomial logit scale factors	(32.66, 27.45, 25.55)
Allowable amounts of backordered demand	(15, 15, 0)

We generate 15 Poisson random variates standing for the number of potential customers in the market for each period of the selling horizon and form 100 demand subscenarios in the forward step as explained in the previous section to obtain a lower bound for the optimal value of the actual SAA problem. By the stopping criterion we choose, the termination of the algorithm necessitates ten percent of the absolute value of the lower bound being larger than the difference between the upper bound and the lower bound.

Firstly, we observe how the manufacturer reacts to an increase in the reimbursement rate to ease its possible negative impact on the expected total profit. For this purpose, we implement the SDDP algorithm by setting the discount rate to 40%, the markdown rate to 20% and the refunds per returned product at the end of the first, second and third period to 300$, 300$ and 100$, respectively. As can be seen in Table 3.3, an increase in the reimbursement rate triggers a decrease in the approximately optimal price the manufacturer should set for the first period. The manufacturer tends to raise the independent retailer's sales volume by increasing its market share as a precautionary measure against high reimbursement costs. The decrease in the retail price also induces an increase in the manufacturer-controlled retailer's market share and in this way; the manufacturer tries to compensate for the reduction in the revenue per product sold.

Table 3.3 The change of the approximate optimal price and market shares in relation to reimbursement rate

Reimbursement rate (%)	10	30	50	70	90
Approximate optimal price ($)	207,90	199.52	194.76	178.31	163.29
Proportion of lost customers (%)	21.79	18.32	16.55	11.45	8.04
Manufacturer's market share (%)	17.11	18.59	19.43	22.25	24.73
Independent retailer's market share (%)	61.10	63.09	64.02	66.30	67.23

In order to make comparisons between some alternative values of the reimbursement rate for the ideal selection of its value, we form two-sided confidence intervals with a confidence level of 80% for the retailers' true expected total profits given the approximately optimal price in the first period. For that purpose, we sequentially solve the models a number of times as in the forward step of the algorithm and generate two Poisson random variates standing for the amounts of demand observed by the retailers between consecutive periods. Before the generation of the Poisson random variates for a given period, the retailers' market shares are computed given the approximately optimal price for that period. In this way, we form 100 demand subscenarios to determine the confidence intervals. The trend shown by the lower and upper bounds of the confidence intervals is shown in Figure 3.1.

As can be seen in Figure 3.1, the manufacturer's true expected total profit shows a downward trend as the reimbursement rate increases. For small values of reimbursement rate, the decrease in the bounds is precipitous as the bounds are almost level for high values of reimbursement rate. The bounds over the independent retailer's true expected total profit increase until the reimbursement rate reaches a certain point, then they start falling slightly. For the manufacturer, the reimbursement rate of 10% dominates all the other alternatives as a reimbursement rate of 70% almost dominates all the other alternatives for the independent retailer. For the selection of an ideal value for the reimbursement rate, the retailers should specify their own minimum allowable expected total profit values. If the manufacturer's minimum allowable profit is less than 1500$, then setting the reimbursement rate to 70% seems to be the best option to lure the independent retailer into keeping the inventory of the corresponding product. Otherwise, some smaller values have to be considered without lessening the independent retailer's true expected total profit below its minimum allowable value. If there exists no feasible solution, then the other contractual parameters have to be fixed at different values.

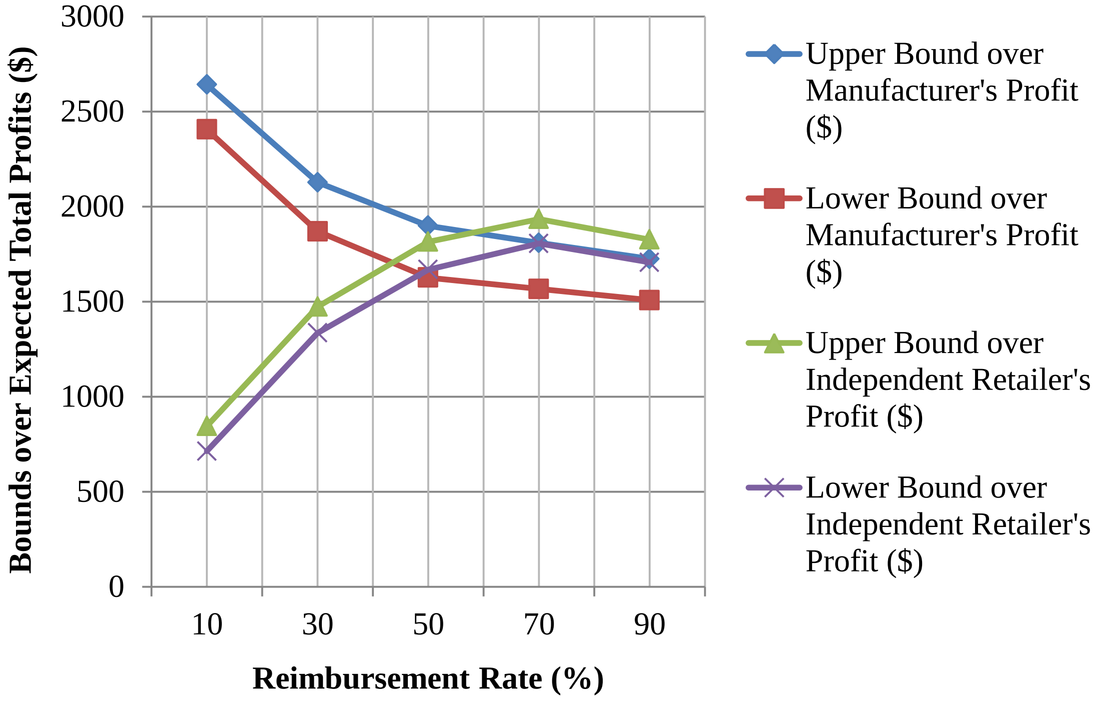

Figure 3.1 The evolution of the bounds over the retailers' expected total net profits in relation to reimbursement rate

Secondly, we observe how the markdown rate impacts the approximately optimal price in the first period and the retailers' true expected total profits exactly in the same way as done for the reimbursement rate. For this purpose, we implement the SDDP algorithm by setting the discount rate to 50%, the reimbursement rate to 70%, the size of the independent retailer's first replenishment order to 30 units and the refunds per returned product at the end of the first, second and third period to 300$, 300$ and 100$, respectively. Since more potential customers are enticed into buying the product from the independent retailer in case the price remains the same as the markdown rate increases, the manufacturer tends to boost the retail price dramatically to mitigate the negative influence of the diminishing market share. As can be seen in Table 3.4, as markdown rate rises, the approximately optimal price in the first period soars.

In order to evaluate different values of the markdown rate in terms of their capabilities of ensuring the manufacturer's and the independent retailer's acceptable profitability levels, we construct two-sided confidence intervals for the retailers' true expected total profits as done for the reimbursement rate. The evolution of the bounds of the confidence intervals in proportion to the markdown rate is presented in Figure 3.2. As can be seen in Figure 3.2, although the independent retailer's market share increases with the markdown rate going up, high values of markdown rate hurt the independent retailer financially because the retail price at which it sells the product diminishes concurrently. The trend shown by the bounds over the manufacturer's expected total profit seems to be erratic. The markdown rate of 10% almost dominates all the other alternatives for the manufacturer as the markdown rate of 0% is apparently the best option for the independent retailer. Seemingly, a markdown rate around 10% is much more preferable for both parties. However, if the manufacturer's minimum allowable expected total profit is less than 1160$, then the markdown rate can be set to 0% to tempt the independent retailer more into keeping the inventory of the product.

Table 3.4 The change of the approximate optimal price and market shares in relation to markdown rate

Markdown rate (%)	0	10	20	30	40
Approximate optimal price ($)	154.65	160.08	164.31	177.44	202.87
Proportion of lost customers (%)	11.08	10.07	8.24	7.59	7.75
Manufacturer's market share (%)	44.46	34.16	24.57	15.14	7.10
Independent retailer's market share (%)	44.46	55.77	67.20	77.27	85.15

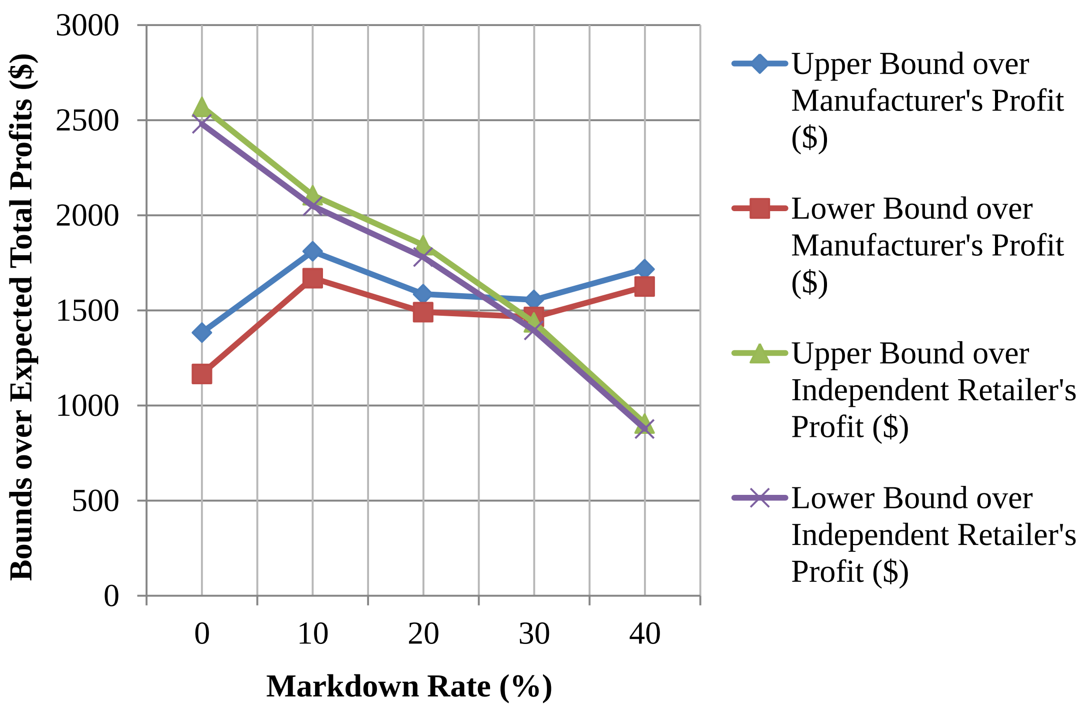

Figure 3.2 The evolution of the bounds over the retailers' expected total net profits in relation to markdown rate

Thirdly, we examine and interpret the influence of the changes in the discount rate on the approximately optimal price in the first period and the retailers' true expected total profits exactly in the same way as done for the preceding parameters. For this purpose, we implement the SDDP algorithm by setting the markdown rate to 20%, the reimbursement rate to 70%, the size of the independent retailer's first replenishment order to 30 units and the refunds per returned product at the end of the first, second and third period to 300$, 300$ and 100$, respectively. As can be seen in Table 3.5, the manufacturer is inclined to reduce the retail price as the discount rate increases. The approximately optimal price is around 186$. If the discount rate is raised from 30% to 70% and the price is not updated, then this implies that the independent retailer purchases the product from the manufacturer at approximately 56$. If the price is also reduced from 186$ to 135$, then this means that the wholesale price is around 41$. That is, the manufacturer earns around 15$ less per product sold to the independent retailer. However, the decrease in the retail price induces an increase in the retailers' market shares. That is, the manufacturer entices more potential customers and drives the independent retailer to make replenishment orders of a larger size. In this way, the manufacturer tries to attenuate the negative impact of the increasing discount rate.

Table 3.5 The change of the approximate optimal price and market shares in relation to discount rate

Discount rate (%)	30	40	50	60	70
Approximate optimal price ($)	186.03	177.69	164.31	157.45	134.77
Proportion of lost customers (%)	13.65	11.29	8.23	6.98	3.97
Manufacturer's market share (%)	20.94	22.35	24.57	25.68	29.26
Independent retailer's market share (%)	65.41	66.36	67.20	67.34	66.77

The evolution of the bounds over the retailers' true expected total profits, determined in the same way as done for the previously examined contractual parameters, in proportion to the discount rate is presented in Figure 3.3. As can be seen from Figure 3.3, the manufacturer's expected total profit is negatively correlated with the discount rate and the manufacturer's expected total profit is traded off against the independent retailer's expected total profit. The discount rate of 70% is the best option among the alternatives for the independent retailer as the discount rate of 30% is the most preferable option for the manufacturer. Therefore, in the selection process for an ideal value of the discount rate, the retailers' minimum allowable profit values play a pivotal role. For example, if an expected total profit value less than 1500$ is undesirable for both retailers, then a discount rate of 50% or slightly less than 50% is the best option. However, if the independent retailer agrees an expected total profit around 1300$, then it is better to set the discount rate to 40%.

The last contractual parameters to be analyzed are the refunds per product in case of mid-life returns and end-of-life returns. In order to observe the impact of the changes in the refund per product returned by the independent retailer between the periods on the approximately optimal price in the first period and the retailers' true expected total profits, we focus on the returned products at the end of the first period. For this purpose, we implement the SDDP algorithm by setting the markdown rate to 20%, the reimbursement rate to 70%, the discount rate to 40%, the size of the independent retailer's first replenishment order to 40 units and the refunds per returned product at the end of the second and third period to 300$ and 100$, respectively. As can be seen from Table 3.6, the manufacturer increases the retail price as the refund per returned product increases. There are no drastic changes in the retailers' market shares. The manufacturer just tries to earn more by selling the product to the end customer at a higher price to compensate for possible reimbursements for the returned products.

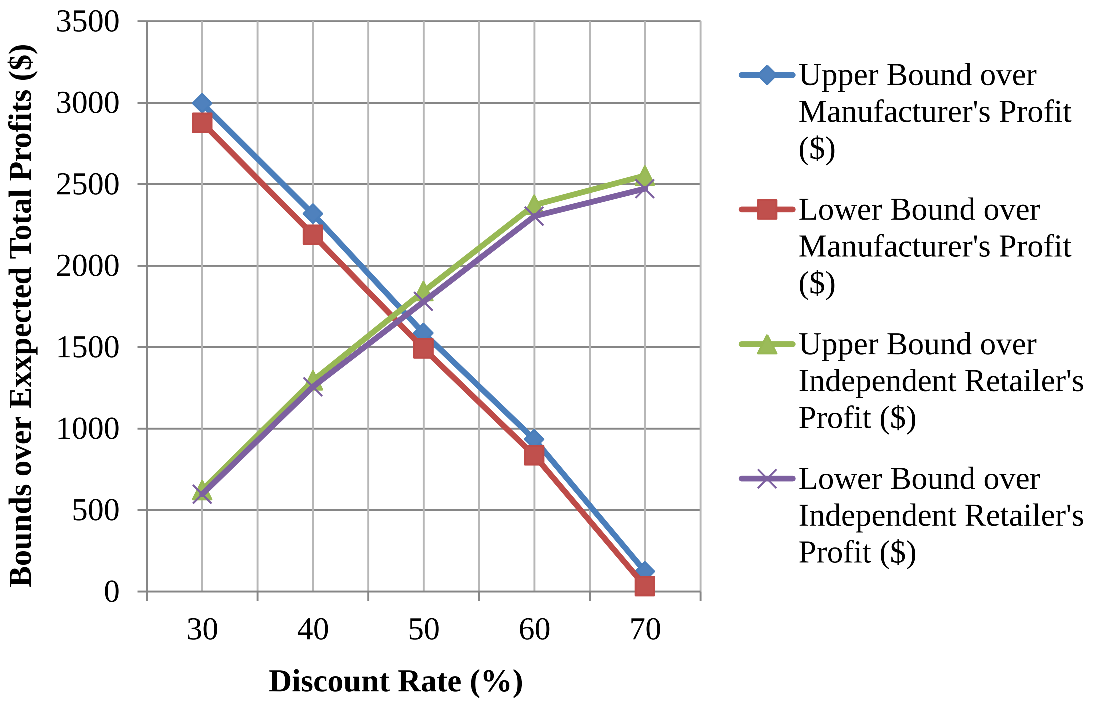

Figure 3.3 The evolution of the bounds over the retailers' expected total net profits in relation to discount rate

Table 3.6 The change of the approximate optimal price and market shares in relation to refund per returned product at the end of the first period

Refund ($)	100	200	300	400	500
Approximate optimal price ($)	175.59	177	178.31	179.54	180.70
Proportion of lost customers (%)	10.75	11.11	11.45	11.78	12.10
Manufacturer's market share (%)	22.71	22.47	22.25	22.04	21.84
Independent retailer's market share (%)	66.54	66.42	66.30	66.18	66.06

The evolution of the bounds over the retailers' true expected total profits in proportion to the refund per product returned by the independent retailer at the end of the first period is presented in Figure 3.4. As can be inferred from Figure 3.4, any increase in the refund afflicts the manufacturer's profitability as higher refunds per product mean higher profitability for the independent retailer. A refund of 100$ per returned product is the best alternative for the manufacturer but a refund of 500$ is the best option for the independent retailer. Just as in the case of discount rate, the retailers' minimum allowable profit values are of great importance. The changes in the refund per product returned by the independent retailer at the end of the second period have a similar impact on the approximately optimal retail price in the first period and the retailers' true expected total profits.

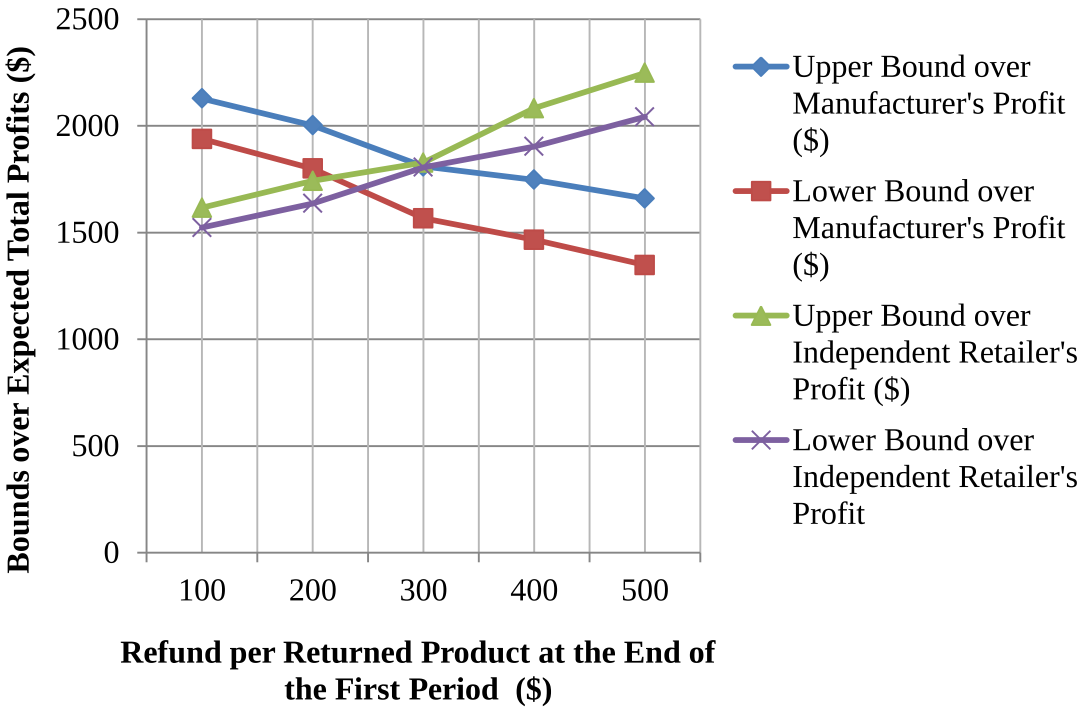

Figure 3.4 The evolution of the bounds over the retailers' expected total net profits in relation to refund per returned product at the end of the first period

In order to analyze the impact of the changes in the refund per product returned at the end of the selling horizon, we implement the SDDP algorithm by setting the markdown rate to 20%, the reimbursement rate to 70%, the discount rate to 40%, the size of the independent retailer's first replenishment order to 40 units and the refunds per returned product at the end of the first and second period to 300$. As shown in Table 3.7, the manufacturer increases the retail price for the same reasons as the refund per product increases.

Table 3.7 The change of the approximate optimal price and market shares in relation to refund per returned product at the end of the selling horizon

Refund ($)	100	200	300	400	500
Approximate optimal price ($)	178.31	178.99	179.63	180.25	180.84
Proportion of lost customers (%)	11.45	11.63	11.81	11.97	12.14
Manufacturer's market share (%)	22.25	22.14	22.02	21.92	21.82
Independent retailer's market share (%)	66.30	66.23	66.17	66.11	66.04

The evolution of the bounds over the retailers' true expected total profits in relation to the refund per product returned by the independent retailer at the end of the selling horizon is presented in Figure 3.5. As can be seen from Figure 3.5, the manufacturer's profitability is negatively correlated with the refund and the manufacturer's profitability is traded off for the independent retailer's profitability. A refund of 100$ per returned product is the best option among the alternatives for the manufacturer but a refund of 500$ is the most preferable alternative for the independent retailer. At each value of the refund between 100$ and 500$, the independent retailer is expected to make higher profit than the manufacturer does. If the manufacturer stipulates making higher profit throughout the selling horizon, then either the refund has to be decreased to a certain level or the values of the other contractual parameters have to be reviewed. The refund has to be selected taking the retailers' minimum allowable profit values into account just as done for the other parameters. If the manufacturer wants to make an expected profit higher than 1500$, then the refund has to be less than 100$. However, if an expected profit larger than 1000$ is sufficient for the manufacturer, then setting the refund to a value around 300$ lures the independent retailer into being involved in such a collaboration. If no feasible solution exists, then the values of the other parameters have to be updated.

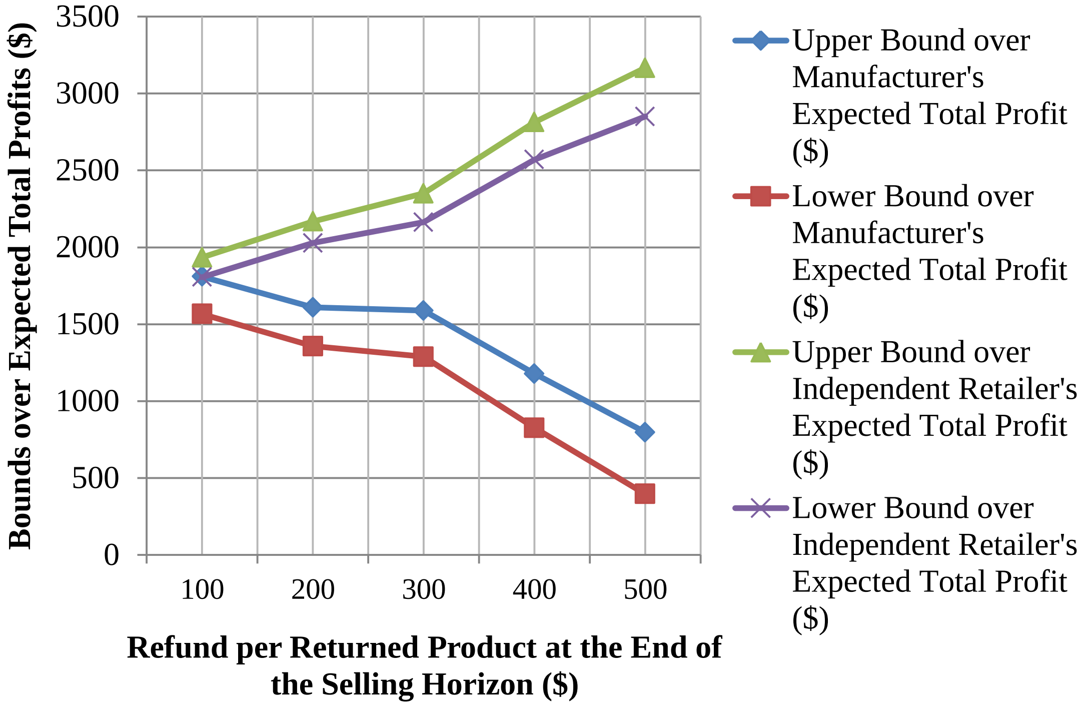

Figure 3.5 The evolution of the bounds over the retailers' expected total net profits in relation to refund per returned product at the end of the selling horizon

The findings on the impact of an increase in the value of each critical contractual parameter on approximately optimal price and the retailers' true expected total net profits are summarized in Table 3.8 shown below.

In conclusion, the results obtained by implementing the variant SDDP algorithm seem logical and the convergence of the algorithm entails a favorably small number of iterations. We can deduce that the retailers' minimum allowable profit values are determinants of the best compromise values for the contractual parameters. Moreover, depending on which retailer's profitability is more highly weighted, the best compromise values can change. The retailers' true expected total profits are very sensitive to the refund per product returned at the end of the selling horizon. End-of-life returns are very effective in inducing the independent retailer to get involved in this business. However, mid-life returns do not seem as effective as end-of-life returns although it is still capable of increasing the profitability of the business for the independent retailer. The selection of ideal values for markdown rate, discount rate and reimbursement rate is very critical because the retailers' expected total profits are very sensitive to the values of these contractual parameters. The values for discount rate and markdown rate should be specified simultaneously. The approximately optimal price that the manufacturer should set in the first period of the selling horizon is also sensitive to the changes in reimbursement rate, discount rate and markdown rate.

Table 3.8 Impact of an increase in the values of the critical contractual parameters on approximately optimal price and the retailers' true expected total net profits

	Approximately optimal price	Confidence interval of manufacturer's expected total net profit	Confidence interval of independent retailer's expected total net profit
Reimbursement rate	Decreasing	Shifting down	First shifting up, then shifting down
Markdown rate	Increasing	First shifting up, then staying almost level	Shifting down
Discount rate	Decreasing	Shifting down	Shifting up
Refund at the end of the first period	Increasing	Shifting down	Shifting up
Refund at the end of the selling horizon	Increasing	Shifting down	Shifting up

Chapter 4

Periodic-review Approximately Optimal Pricing in the Presence of Price Protection, End-of-life Return Opportunity and a Special Discount Policy

In this chapter, firstly, the differences in the problem definition compared to the problem discussed in Chapter 2 are provided by defining the boundaries of the research built on some assumptions. Secondly, the stochastic programming model to be solved to determine the manufacturer's optimal pricing strategy is presented and it is discussed whether any changes are needed in the implementation of the variant SDDP algorithm proposed in Chapter 2. Finally, the results of the numerical experiments carried out to observe how the changes in some contractual parameters playing a significant role in the manufacturer's pricing decisions impact the approximate optimal price, the retailers' market shares and their true expected total net profits are presented.

4.1 Problem Definition

In this chapter, we include a special discount policy in the price commitment policy discussed in Chapter 2 as a supplementary privilege offered to the independent retailer by the manufacturer. In this case, the discount rate that the manufacturer offers the independent retailer for its replenishment orders depends on order size. As order size increases, the discount rate applied on the retail price set by the manufacturer increases. Moreover, the same discount rate is valid for every single product in a replenishment order. That is, if the independent retailer orders in a larger batch, it is entitled to pay a lower wholesale price per product. However, the independent retailer might stipulate a minimum level for the discount rate and likewise, the manufacturer might ask for a maximum level.

4.2 Model and Methodology

The models constructed for the price commitment policy discussed in Chapter 2 has to be adapted to the new price commitment policy. Table 4.1 provided below presents the notation in this section.

Table 4.1 Notation

$\boldsymbol{R}$	Inventory holding cost per dollar per period
$\boldsymbol{\theta}$	Discount rate per product backordered
$\boldsymbol{\beta(o)}$	Discount rate function of order size per product ordered by the independent retailer
$\boldsymbol{\alpha}$	Reimbursement rate
$\boldsymbol{c_t}$	Production cost per product in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t^m}$	Demand observed by the manufacturer-controlled retailer in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t^r}$	Demand observed by the independent retailer in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t}$	Ordered pair (D_t^m, D_t^r) of demands observed by retailers in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{N_t^m}$	Allowable number of backordered products for manufacturer-controlled retailer in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{N_t^r}$	Allowable number of backordered products for independent retailer in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{S_t^m}$	Manufacturer-controlled retailer's order-up-to level in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{S_t^r}$	Independent retailer's order-up-to level in period *t* $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{I_t^m}$	Manufacturer-controlled retailer's on-hand inventory level right before observing demand in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{I_t^r}$	Independent retailer's on-hand inventory level right before observing demand in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{w_m}$	Salvage value of manufacturer-controlled retailer's unsold inventory at the end of the selling horizon
$\boldsymbol{w_r}$	Refund per product returned by the independent retailer at the end of the selling horizon
$\boldsymbol{p_t}$	Retail price set by the manufacturer in period t $\forall t \in \{1,2,\dots N\}$

Given the assumption that the retailers start the selling horizon with no on-hand stock, they both place a replenishment order at the beginning of the first period. The manufacturer-owned retailer (m)'s and the independent retailer (r)'s post-replenishment inventory levels are their order-up-to levels S_1^m and s_1^r in the first period, respectively. The manufacturer incurs some production cost because of these replenishment orders.

The manufacturer has to set the retail price (p_1) for the first period to determine the wholesale price ($p_1.\left(1-\beta(s_1^r)\right)$) and the retail price ($p_1.(1-\gamma)$) offered by the independent retailer in compliance with RFM policy. The revenue generated from the independent retailer's replenishment order and the revenue generated by selling the product to the end customer make up the manufacturer's total revenue in the first period. The former revenue is directly generated after the satisfaction of the independent retailer's replenishment order and it can be called post-replenishment revenue. As the former revenue is deterministic, the latter is random because it depends on the random demand observed by the manufacturer-owned retailer in the first period.

The post-decision profit-to-go function ($\varphi_1(S_1^m, s_1^r, p_1)$) encompasses the expected revenue generated from the products sold to the end customer in the first period, the expected revenue generated by satisfying the backordered demand in the second period and the optimal expected profit made from the second period till the end of the selling horizon. Since the retail price is assumed to decline over time because of the obsolescence of the product, the price set in the first period has an influence on the manufacturer's pricing decision in the second period. Just as in the first period, the manufacturer also takes the retailers' pre-replenishment inventory levels into consideration when specifying the retail price in the second period. Therefore, the post-decision profit-to-go function of the first period is a function of the post-replenishment inventory levels (S_1^m and s_1^r) and the retail price (p_1) set in the first period.

The model that has to be solved to determine the optimal retail price that the manufacturer should set in the first period of the selling horizon is as follows:

$$\max_{p_1 \in A_1} -S_1^m.c_1 + \left(\left(1-\beta(S_1^r)\right).p_1 - c_1\right).S_1^r + \varphi_1(S_1^m, S_1^r, p_1), \tag{4.1}$$

where

$$A_1 = \{p_1 \in \mathbb{R}: p_1 \geq 0\}, \tag{4.2}$$

$$\varphi_1(S_1^m, S_1^r, p_1) = \mathbb{E}_{D_1}[Q_2(S_1^m, S_1^r, p_1, D_1)]. \tag{4.3}$$

In order to determine the optimal retail price for a given intermediate period *t*, the manufacturer should maximize the expected total profit made from that period till the end of the selling horizon. The pre-decision profit-to-go function ($Q_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1})$) of a given intermediate period *t* is derived by adding the revenue generated from the products sold to the end customer in period *t-1* and the revenue generated in period *t* by satisfying the demand backordered in period *t-1* to the expected total profit made from period *t* till the end of the selling horizon. Then, the pre-decision profit-to-go function of period t can be characterized by the optimal value of the following mathematical model:

$$
\begin{aligned}
Q_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}) = \max_{p_t \in A_t} & - max\{min\{D_{t-1}^m - I_{t-1}^m, N_{t-1}^m\}, 0\}\,.\,\theta.\, p_{t-1} + \\
& + min\{D_{t-1}^m, I_{t-1}^m + N_{t-1}^m\}.\, p_{t-1} - \\
& - max\{I_{t-1}^m - D_{t-1}^m, 0\}.\, c_{t-1}.\, r - \\
& - max\{S_t^m - I_{t-1}^m + D_{t-1}^m, 0\}.\, c_t + \\
& + max\{D_{t-1}^m - I_{t-1}^m - N_{t-1}^m, 0\}.\, c_t + \\
& + min\{S_t^r + N_{t-1}^r, max\{S_t^r - I_{t-1}^r + D_{t-1}^r, 0\}\}. \\
.\Big(\big(1 - \beta\big(min\{S_t^r + N_{t-1}^r, max\{S_t^r - I_{t-1}^r + D_{t-1}^r, 0\}\}\big)\big)&.\, p_t - c_t\Big) - \\
& - max\{I_{t-1}^r - D_{t-1}^r, 0\}.\, \alpha.\, (p_{t-1} - p_t) \\
& + \varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m)\,, IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t), \qquad (4.4)
\end{aligned}
$$

where

$$A_t = \{p_t \in \mathbb{R}: p_t \le p_{t-1}, p_t \ge 0\}, \qquad (4.5)$$

$$\varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m)\,, IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t) = \mathbb{E}_{D_t}[Q_{t+1}(IL_t^m(I_{t-1}^m, D_{t-1}^m)\,, IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t, D_t)], \qquad (4.6)$$

$$IL_t^m(I_{t-1}^m, D_{t-1}^m) \ = \ max\{S_t^m, I_{t-1}^m - D_{t-1}^m\}, \qquad (4.7)$$

$$IL_t^r(I_{t-1}^r, D_{t-1}^r) \ = \ max\{S_t^r,\ I_{t-1}^r - D_{t-1}^r\}. \qquad (4.8)$$

The profit function of the dummy period $N+1$ given the retailers' inventory levels before observing demand in period N, demand observed by the retailers in period N and the price set by the manufacturer in period N is as follows:

$$
\begin{aligned}
Q_{N+1}(I_N^m, I_N^r, p_N, D_N) \ = \ & - max\{I_N^m - D_N^m, 0\}.\, c_N.\, r + D_N^m.\, p_N - \ max\{D_N^m - I_N^m, 0\}.\, p_N + \\
& + max\{I_N^m - D_N^m, 0\}.\, w_m + \ max\{I_N^r - D_N^r, 0\}.\, (w_m - w_r). \qquad (4.9)
\end{aligned}
$$

The way how the variant SDDP algorithm is implemented for this price commitment policy is the same as the way how it is implemented for the price commitment policy discussed in Chapter 2.

4.3 Numerical Experiment

As we did for the previous price commitment contracts, we observe how the changes in some contractual parameters impact the approximately optimal price at which the manufacturer- controlled retailer should sell the product in the first period of the selling horizon of three periods and the retailers' expected total true profits. In accordance with the special discount policy that the manufacturer offers the independent retailer, the discount applied on the independent retailer's replenishment order in any period is relative to the size of the order. In the previous section, we assume a special discount rate function to determine the discount rate given the size of the independent retailer's replenishment order in any period. The discount rate function employed for the analysis in this section is as follows:

$$\beta(s) = 1 - \min\{1 - \beta_{min}, \max\{1 - \beta_{max}, \rho^{-s}\}\}. \tag{4.10}$$

As can be seen in the function shown above, the discount rate is bounded by the minimum allowable discount rate β_{min} requested by the independent retailer and the maximum allowable discount rate β_{max} stipulated by the manufacturer. The parameter ρ stands for the compromise discount factor. The retailers' price-dependent market shares are determined by the MNL model explained in Section 2.4.

The contractual parameters examined in this section are minimum allowable discount rate, maximum allowable discount rate, discount factor, markdown rate, refund per product returned by the independent retailer at the end of the selling horizon and reimbursement rate. Throughout the section, the values of some parameters are mostly fixed and unless otherwise stated, they are as shown in Table 4.2 provided below.

Table 4.2 The values of fixed contractual and non-contractual parameters

Parameter	Value
Holding cost per dollar per period ($)	0.05
Discount rate for backordered demand (%)	15
Salvage value at the end of the selling horizon ($)	60
Production costs ($)	(60, 60, 60)
Manufacturer-controlled retailer's order-up-to levels	(22, 19, 17)
Independent retailer's order-up-to levels	(20, 18, 15)
Mean number of potential customers in periods	(22, 19, 15)
Mean maximum-willingness-to-pay values for retailers ($)	(200, 175, 140)
Multinomial logit scale factors	(32.66, 27.45, 25.55)
Allowable amounts of backordered demand for retailers	(15, 15, 0)

We generate 15 Poisson random variates standing for the number of potential customers in the market for each period of the selling horizon and form 100 demand subscenarios in the forward step as explained in Section 2.3 to obtain a lower bound for the optimal value of the actual SAA problem. By the stopping criterion we choose, the termination of the algorithm necessitates ten percent of the absolute value of the lower bound being larger than the difference between the upper bound and the lower bound.

Firstly, we evaluate the manufacturer's response to an increase in the minimum allowable discount rate to alleviate its likely adverse effect on its expected total profit. For this purpose, we implement the SDDP algorithm by setting the discount factor to 1.03, the maximum discount rate to 50%, the markdown rate to 20%, the reimbursement rate to 70% and the refund per returned product to 200$, respectively. As can be seen in Table 4.3, an increase in the minimum allowable discount rate increases the approximately optimal price that the manufacturer should set in the first period. The reason underlying the manufacturer’s tendency to raise the retail price is to attenuate the negative influence of the offered extra discount on its expected total profit. However, since the manufacturer-controlled retailer’s market share and the expected size of the replenishment order that can be placed by the independent retailer in the second period reduce as the retail price increases, the manufacturer shows a conservative behavior on the price. Therefore, we do not see a surge in the retail price. Another reason why the manufacturer refrains from diminishing the independent retailer’s market share dramatically is to avoid a high reimbursement cost.

Table 4.3 The change of the approximate optimal price and market shares in relation to the minimum allowable discount rate

Minimum allowable discount rate (%)	25	30	35	40	45
Approximate optimal price ($)	173.55	173.99	174.97	175.67	176.36
Proportion of lost customers (%)	10.25	10.36	10.60	10.77	10.95
Manufacturer’s market share (%)	23.05	22.97	22.81	22.69	22.58
Independent retailer’s market share (%)	66.70	66.67	66.59	66.54	66.47

As can be seen in Figure 4.1, an increase in the minimum allowable discount rate induces a downward trend in the bounds over the manufacturer’s true expected total profit. On the contrary, the confidence interval of the independent retailer’s true expected total profit shifts upward as the minimum allowable discount rate increases. The minimum allowable discount rate of 25% is the most preferable one among all the evaluated alternatives for the manufacturer as the minimum allowable discount rate of 45% dominates all the other alternatives for the independent retailer. If the minimum allowable discount rate is fixed at a specific value between 25% and 30%, the confidence intervals of the retailers’ true expected total profits almost completely overlap. If a true expected total profit below 2000$ is undesirable for the manufacturer, then the two parties need to bargain for a minimum allowable discount rate lower than 25% or play around with different values of the other fixed contractual parameters. Of course, the minimum allowable true expected total profit that the independent retailer asks for also has to be reckoned with. If a true expected total profit above 1500$ is satisfactory for the manufacturer, then setting the minimum allowable discount rate to 35% seems to be the best option to tempt the independent retailer into keeping the inventory of the product.

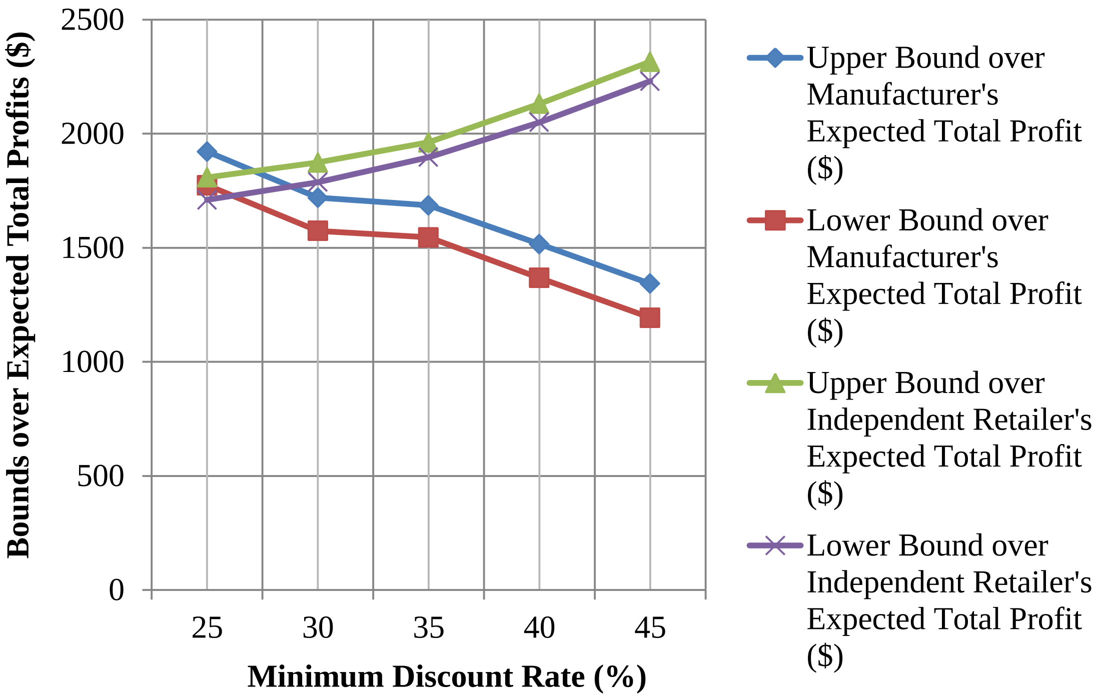

Figure 4.1 The evolution of the bounds over the retailers' expected total net profits in relation to the minimum allowable discount rate

Secondly, we observe how the maximum allowable discount rate impacts the approximately optimal price in the first period and the retailers' true expected total profits. For this purpose, we implement the SDDP algorithm by setting the discount factor to 1.03, the minimum discount rate to 25%, the markdown rate to 20%, the reimbursement rate to 70% and the refund per returned product to 200$, respectively. As can be seen in Table 4.4 shown below, the manufacturer tends to decrease the retail price to raise the retailers' market shares as the maximum allowable discount rate goes up till it reaches 50%. The reason why the manufacturer maintains 173.55$ as the retail price for the maximum allowable discount rates above 50% is the relatively low demand observed for slow-moving A items. That is, the probability of the independent retailer observing a higher demand than a specific amount is negligibly small so the extra discount offered by increasing the maximum allowable discount rate has almost no effect on the manufacturer's true expected total profit. This fact can be observed in Figure 4.2, as well. If the maximum allowable discount rate is increased above 50%, the bounds over the retailers' true expected total profits remain almost the same.

Table 4.4 The change of the approximate optimal price and market shares in relation to maximum allowable discount rate

Maximum allowable discount rate (%)	30	40	50	60	70
Approximate optimal price ($)	185.40	177.03	173.55	173.55	173.55
Proportion of lost customers (%)	13.46	11.12	10.25	10.25	10.25
Manufacturer's market share (%)	21.04	22.46	23.05	23.05	23.05
Independent retailer's market share (%)	65.50	66.42	66.70	66.70	66.70

As can be seen in Figure 4.2, as the maximum allowable discount rate increases, the confidence interval of the manufacturer's true expected total profit shifts downward till the rate reaches 50% and then the graph flattens out. On the contrary, the confidence interval of the independent retailer's true expected total profit shifts upward and the intervals start overlapping after the maximum allowable discount rate exceeds 50%. That is, the confidence intervals of the retailers' expected total profits show a similar behavior as the maximum allowable discount rate keeps increasing above 50%. Furthermore, the retailers' true expected total profits are almost level if the maximum allowable discount rate is set to a value higher than or equal to 50%. The maximum allowable discount rate of 50% is one of the most preferable alternatives for the independent retailer as the maximum allowable discount rate of 30% dominates all the other alternatives for the manufacturer. If a true expected total profit above 1500$ is satisfactory for the independent retailer, then it is the best option to set the maximum allowable discount rate to 40% for the manufacturer to attain the highest true expected total profit possible. Of course, the minimum allowable true expected total profit that the manufacturer asks for also has to be taken into account. If a true expected total profit above 1500$ is satisfactory for the manufacturer, then setting the maximum allowable discount rate to 50% seems to be the best alternative to entice the independent retailer into keeping the inventory of the product. If there does not exist any compromise value of the maximum discount rate that does not violate the true expected total profit limits, then different values of the other contractual parameters should be negotiated.

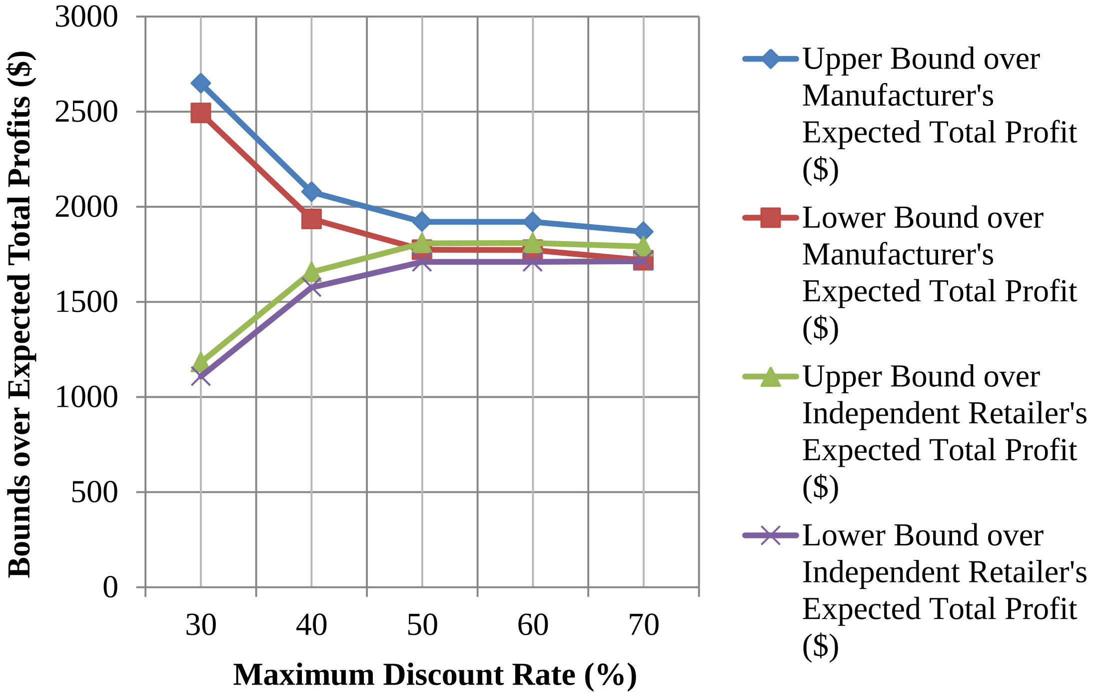

Figure 4.2 The evolution of the bounds over the retailers' expected total net profits in relation to maximum allowable discount rate

The next contractual parameter that we examine is the discount factor. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 50%, the minimum allowable discount rate to 25%, the markdown rate to 20%, the reimbursement rate to 70% and the refund per returned product to 200$, respectively. As can be seen in Table 4.5, the manufacturer should pull down the retail price to increase the retailers' market shares as the discount factor increases. In this way, the manufacturer drives the independent retailer to place a greater amount of replenishment order in the second period and lures more potential customers to purchase the product from the manufacturer-controlled retailer. This approach is intended to mitigate the negative impact of the increase in the decreasing rate of the discount rate.

Table 4.5 The change of the approximate optimal price and market shares in relation to discount factor

Discount factor	1	1.02	1.03	1.05	1.07
Approximate optimal price ($)	188.80	183.41	173.55	169.55	169.26
Proportion of lost customers (%)	14.52	12.87	10.25	9.33	9.27
Manufacturer's market share (%)	20.46	21.39	23.05	23.71	23.76
Independent retailer's market share (%)	65.02	65.75	66.70	66.96	66.98

As can be seen in Figure 4.3, an increase in the discount factor pulls down the bounds over the manufacturer's true expected total profit. On the contrary, the confidence interval of the independent retailer's true expected total profit shifts up as the discount factor rises. Obviously, the bounds change more steeply in relation to discount factor than they do in relation to the maximum and minimum allowable discount rates. The retailers' expected total profits are almost the same if the discount factor is around 1.03. The discount factor of 1.07 seems to be the most alluring option among the evaluated alternatives for the independent retailer as the discount factor of 1 is the most appealing one for the manufacturer. The retailers' minimum allowable true expected total profits are decisive in the selection of a compromise value of the discount factor as is the case for the other contractual parameters. For example, if a true expected total profit of 1500$ is sufficient for the independent retailer, then a discount factor which is slightly less than 1.03 is the best preference for the manufacturer to make the highest possible expected total profit. If the manufacturer sets its minimum allowable true expected total profit to 2500$ and the true expected total profit below 1500$ is undesirable for the independent retailer, then there is not any compromise value of the discount factor. In that case, the parties should negotiate different values of the other contractual parameters.

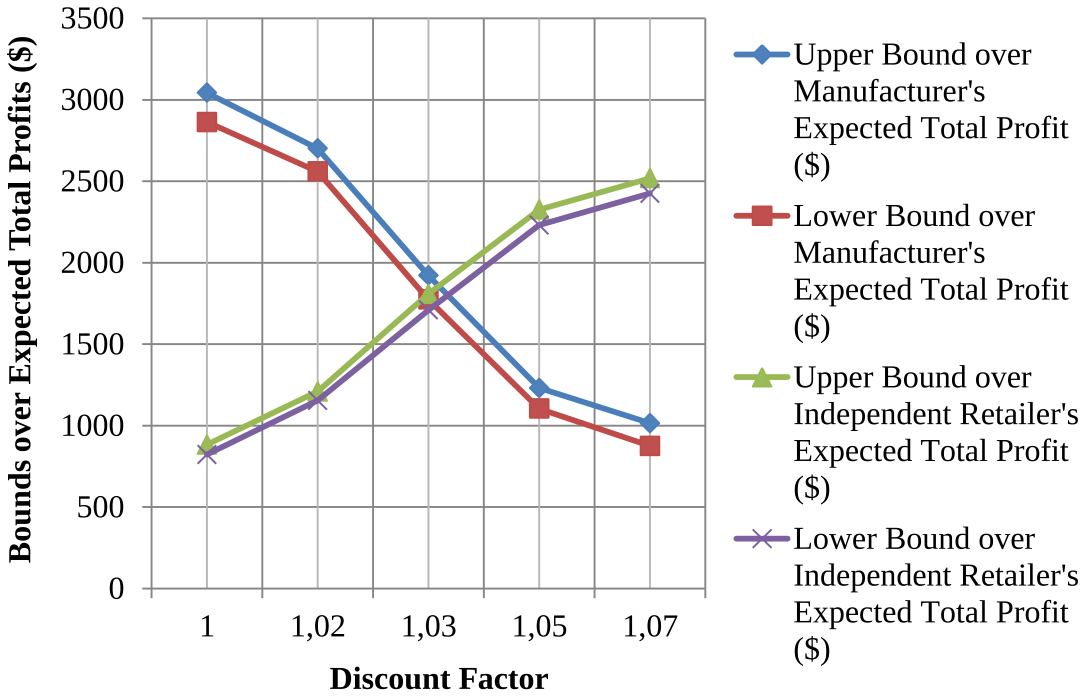

Figure 4.3 The evolution of the bounds over the retailers' expected total net profits in relation to discount factor

Another contractual parameter to be analyzed is the markdown rate. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 50%, the minimum allowable discount rate to 45%, the discount factor to 1.03, the reimbursement rate to 70% and the refund per returned product to 200$, respectively. As shown in Table 4.6, the retail price that the manufacturer should set in the first period soars as the markdown rate increases. It is evident that any increase in the markdown rate renders the independent retailer more appealing to purchase the product from if the price does not change. Therefore, the manufacturer risks reducing its own market share by increasing the price to earn more per product. In that way, the manufacturer tries to compensate for the loss arising from the potential customers who give up purchasing the product from the manufacturer-controlled retailer and prefer the independent retailer instead. Obviously, the retail price increases more precipitously as the markdown rate increases.

Table 4.6 The change of the approximate optimal price and market shares in relation to markdown rate

Markdown rate (%)	0	10	20	30	40
Approximate optimal price ($)	161.95	166.72	175.36	189.74	214.40
Proportion of lost customers (%)	7.78	8.73	10.70	14.83	24.78
Manufacturer’s market share (%)	24.95	24.17	22.74	20.30	15.95
Independent retailer’s market share (%)	67.27	67.10	66.56	64.87	59.27

As can be inferred from Figure 4.4, the confidence interval of the manufacturer’s true expected total profit shifts up as the markdown rate rises. The graph gets steeper when the markdown rate is above 20%. On the contrary, the confidence interval of the independent retailer’s true expected total profit shifts down as the markdown rate increases. As can be seen in Figure 4.4, the confidence interval of the independent retailer’s true expected total profit shifts more dramatically than the confidence interval of the manufacturer’s true expected total profit does. The markdown rate of 0% is the most preferable alternative for the independent retailer but it is dominated by all the other alternatives from the manufacturer’s perspective. The most attractive option for the manufacturer is to set the markdown rate to 40%. However, in that case, the profitability of this collaboration for the independent retailer is seriously weighed on since the independent retailer’s true expected total profit goes down below 1000$. For that reason, the retailers’ minimum allowable expected profit values have a critical role in the selection of a compromise value of the markdown rate. For example, if a true expected total profit above 1000$ is satisfactory for the manufacturer, then the markdown rate can be set to 0% to enable the independent retailer to achieve the highest possible expected total profit. However, if the manufacturer needs an expected total profit around 1500$, then it is the best option to set the markdown rate to 30%. If there is not any compromise value of the markdown rate, then different values of the other contractual parameters should be negotiated or the parties should revise their profit expectations.

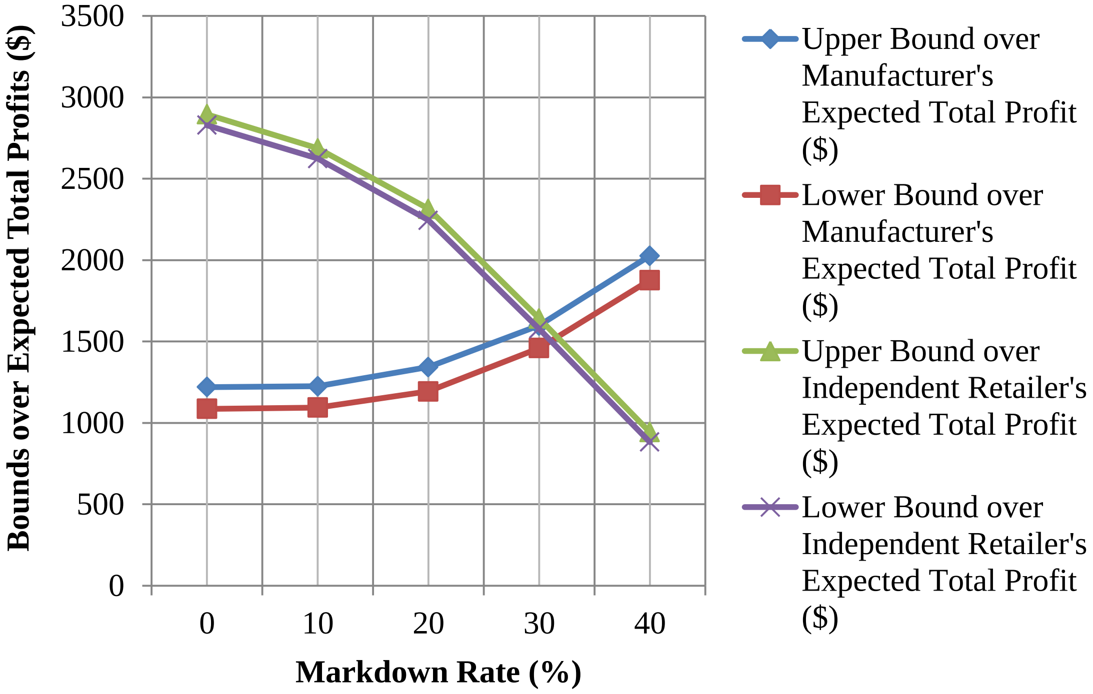

Figure 4.4 The evolution of the bounds over the retailers' expected total net profits in relation to markdown rate

The fifth contractual parameter that we evaluate is the refund per returned product at the end of the selling horizon. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 40%, the minimum allowable discount rate to 25%, the discount factor to 1.03, the markdown rate to 20% and the reimbursement rate to 70%, respectively. As can be seen in Table 4.7, the retail price that the manufacturer should set in the first period almost does not change as the refund per returned product increases. The changes in the approximately optimal retail price emerge starting from the third digit after the decimal point so we can state that any increase in the refund per returned product has no conspicuous effect on the approximately optimal retail price.

Table 4.7 The change of the approximate optimal price and market shares in relation to refund per returned product at the end of the selling horizon

Refund ($)	150	200	250	300	350
Approximate optimal price ($)	177.03	177.03	177.03	177.03	177.03
Proportion of lost customers (%)	11.12	11.12	11.12	11.12	11.12
Manufacturer's market share (%)	22.46	22.46	22.46	22.46	22.46
Independent retailer's market share (%)	66.42	66.42	66.42	66.42	66.42

The confidence interval of the manufacturer's true expected total profit shifts down as the refund per returned product goes up as shown in Figure 4.5. Furthermore, the independent retailer's true expected total profit is negatively correlated with the manufacturer's true expected total profit. A refund of 350$ is the most tempting option for the independent retailer among all the assessed alternatives as a refund of 150$ is the most attractive one for the manufacturer. Just like for the previously analyzed contractual parameters, the retailers' minimum allowable true expected profits play a pivotal role in the determination of a compromise value of the refund per product. For example, if an expected total profit above 1500$ is adequate for the manufacturer, then the refund per returned product can be set to 250$ to lure the independent retailer into keeping the inventory of the product. If an expected total profit above 1500$ is not deficient for the independent retailer, then the best option is to set the refund per product to 200$ for the manufacturer to make the highest possible expected total profit. If there is not any compromise value of the refund per product, then the parties should negotiate different values of the other contractual parameters or revise their profit expectations.

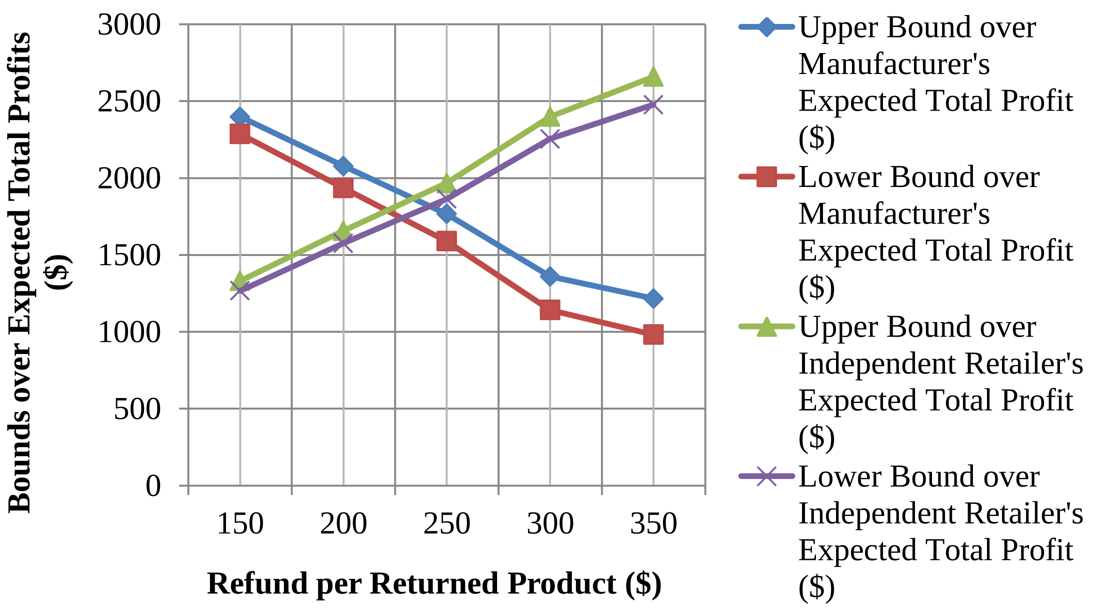

Figure 4.5 The evolution of the bounds over the retailers' expected total net profits in relation to refund per returned product at the end of the selling horizon

The last contractual parameter that we examine is the reimbursement rate. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 40%, the minimum allowable discount rate to 25%, the discount factor to 1.03, the markdown rate to 20% and the refund per returned product to 200$, respectively. The approximately optimal price decreases as the reimbursement rate increases as can be seen in Table 4.8. The manufacturer tends to raise the independent retailer's market share in purpose for diminishing its expected on-hand stock at the end of the first period. Moreover, a decrease in the retail price in the first period limits the amount of reduction in the retail price in the second period. This means that the manufacturer aims to evade high reimbursement costs.

Table 4.8 The change of the approximate optimal price and market shares in relation to reimbursement rate

Reimbursement rate (%)	10	30	50	70	90
Approximate optimal price ($)	182.24	180.65	178.92	177.03	174.94
Proportion of lost customers (%)	12.53	12.08	11.61	11.12	10.59
Manufacturer's market share (%)	21.58	21.85	22.15	22.46	22.81
Independent retailer's market share (%)	65.89	66.07	66.24	66.42	66.60

As can be deduced from Figure 4.6, the confidence interval of the manufacturer's true expected total profit shifts down as the reimbursement rate increases and the independent retailer's true expected total profit is negatively correlated with the manufacturer's true expected total profit. However, the confidence intervals seem to be almost completely overlapping. This means that the price protection does not have a substantial impact on the retailers' true expected total profits when this price commitment policy comprising price protection, end-of-life return and special discount policy is in effect.

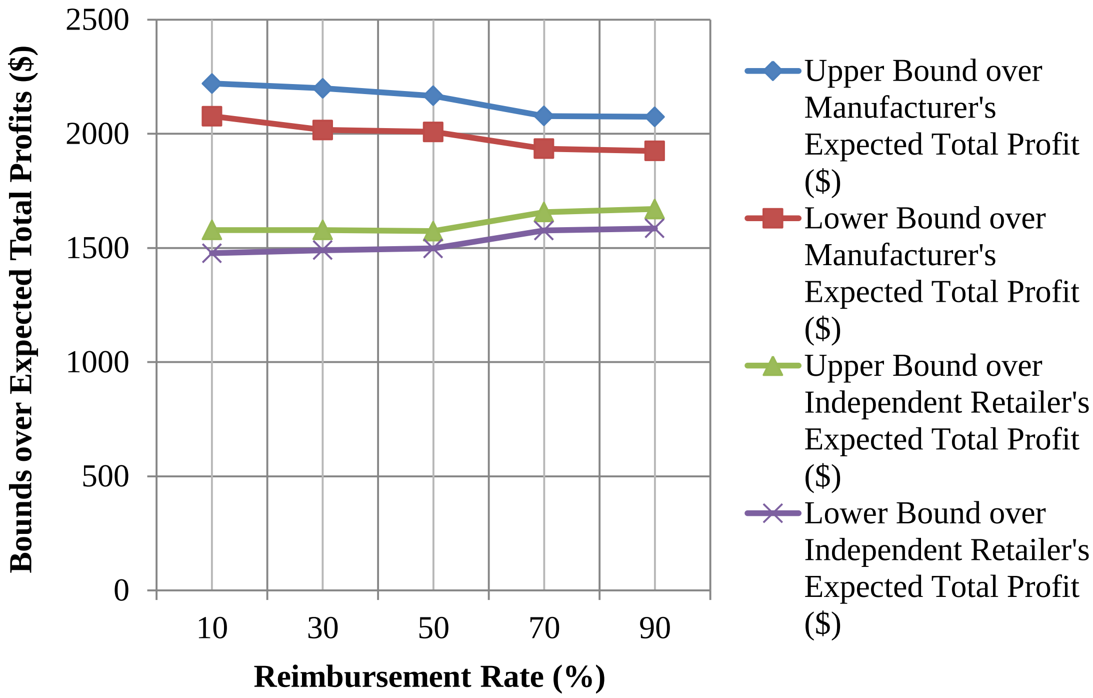

Figure 4.6 The evolution of the bounds over the retailers' expected total net profits in relation to reimbursement rate

The findings on the impact of an increase in the value of each critical contractual parameter on approximately optimal price and the retailers' true expected total net profits are summarized in Table 4.9 shown below.

Table 4.9 Impact of an increase in the values of the critical contractual parameters on approximately optimal price and the retailers' true expected total net profits

	Approximately optimal price	Confidence interval of manufacturer's expected total net profit	Confidence interval of independent retailer's expected total net profit
Minimum allowable discount rate	Increasing	Shifting down	Shifting up
Maximum allowable discount rate	First decreasing, then staying almost the same	First shifting down, then staying almost level	First shifting up, then staying almost level
Discount factor	Decreasing	Shifting down	Shifting up
Markdown rate	Increasing	Shifting up	Shifting down
Refund at the end of the selling horizon	Almost not changing	Shifting down	Shifting up
Reimbursement rate	Decreasing	Almost not shifting	Almost not shifting

In conclusion, the retailers' minimum allowable profit values have a critical role in the selection of the compromise values of each contractual parameter except for the reimbursement rate. The existence of price protection in this price commitment policy is contestable in terms of whether it renders the business more profitable for the independent retailer or not. The mid-life returns included in the price commitment policy examined in Section 3.3 can be substituted by the special discount policy to lure the independent retailer into keeping the inventory of the product since the existence of the special discount policy renders this business more profitable for the independent retailer. Furthermore, the manufacturer has a chance to encourage the independent retailer to place replenishment orders of greater size in presence of special discount policy. This means that the special discount policy is beneficial for both of the parties.

Chapter 5

Periodic-review Approximately Optimal Pricing in the Presence of Price Protection, Mid-life Return Opportunities, End-of-life Return Opportunity and a Special Discount Policy

In this chapter, firstly, the differences in the problem definition compared to the problem discussed in Chapter 2 are provided by defining the boundaries of the research built on some assumptions. Secondly, the stochastic programming model to be solved to determine the manufacturer's optimal pricing strategy is presented and it is discussed whether any changes are needed in the implementation of the variant SDDP algorithm proposed in Chapter 2. Finally, the results of the numerical experiments carried out to observe how the changes in some contractual parameters playing a significant role in the manufacturer's pricing decisions impact the approximate optimal price, the retailers' market shares and their true expected total net profits are presented.

5.1 Problem Definition

In this chapter, we analyze a price commitment policy consisting of all the privileges discussed so far. That is, the manufacturer concurrently offers the independent retailer price protection, mid-life return opportunities, end-of-life return opportunity and the special discount policy discussed in the previous chapters. The details about all the involved privileges are already provided in the previous chapters.

5.2 Model and Methodology

The models constructed for the price commitment policy discussed in Chapter 2 has to be adapted to the new price commitment policy. Table 5.1 provided below presents the notation in this section.

Table 5.1 Notation

$\boldsymbol{R}$	Inventory holding cost per dollar per period
$\boldsymbol{\theta}$	Discount rate per product backordered
$\boldsymbol{\beta(o)}$	The function of discount rate per product ordered by the independent retailer
$\boldsymbol{\alpha}$	Reimbursement rate
$\boldsymbol{c_t}$	Production cost per product in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{D_t^j}$	Demand observed by retailer j in period t $\forall t \in \{1,2,\dots N\}, \forall j \in \{m,r\}$
$\boldsymbol{D_t}$	Ordered pair of demands observed by retailers in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{N_t^j}$	Allowable number of products backordered by retailer j in period t $\forall t \in \{1,2,\dots N\}, \forall j \in \{m,r\}$
$\boldsymbol{S_t^m}$	Manufacturer-controlled retailer's order-up-to level in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{S_t^r}$	Independent retailer's dispose-down-to level in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{s_t^r}$	Independent retailer's order-up-to level in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{w_t^m}$	The value per product salvaged by the manufacturer in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{w_t^r}$	Refund per product returned by the independent retailer in period t $\forall t \in \{1,2,\dots N\}$
$\boldsymbol{I_t^j}$	Retailer j's post-replenishments inventory level in period t $\forall t \in \{1,2,\dots N\}, \forall j \in \{m,r\}$
$\boldsymbol{p_t}$	Retail price set by the manufacturer in period t $\forall t \in \{1,2,\dots N\}$

The model that has to be solved to determine the optimal retail price that the manufacturer should set in the first period of the selling horizon is as follows:

$$\max_{p_1 \in A_1} -S_1^m . c_1 + \left(\left(1-\beta(s_1^r)\right).p_1 - c_1\right).s_1^r + \varphi_1(S_1^m, s_1^r, p_1), \tag{5.1}$$

where

$$A_1 = \{p_1 \in \mathbb{R}: p_1 \geq 0\}, \tag{5.2}$$

$$\varphi_1(S_1^m, s_1^r, p_1) = \mathbb{E}_{D_1}[Q_2(S_1^m, s_1^r, p_1, D_1)]. \tag{5.3}$$

The pre-decision profit-to-go function of a given intermediate period t is equivalent to the optimal value of the mathematical model shown below.

$$
\begin{aligned}
Q_t(I_{t-1}^m, I_{t-1}^r, p_{t-1}, D_{t-1}) = \max_{p_t \in A_t} & - max\{min\{D_{t-1}^m - I_{t-1}^m, N_{t-1}^m\}, 0\}.\theta.p_{t-1} + \\
& + min\{D_{t-1}^m, I_{t-1}^m + N_{t-1}^m\}.p_{t-1} - \\
& - max\{I_{t-1}^m - D_{t-1}^m, 0\}.c_{t-1}.r - \\
& - max\{S_t^m - I_{t-1}^m + D_{t-1}^m, 0\}.c_t + \\
& + max\{D_{t-1}^m - I_{t-1}^m - N_{t-1}^m, 0\}.c_t + \\
+ min\big\{max\{S_t^m - I_{t-1}^m + D_{t-1}^m, 0\} - max\{D_{t-1}^m - I_{t-1}^m - N_{t-1}^m, 0\}, & \max\{I_{t-1}^r - D_{t-1}^r - S_t^r, 0\}\big\}. \\
& .(c_t - w_t^m) - \max\{I_{t-1}^r - D_{t-1}^r - S_t^r, 0\}.(w_t^r - w_t^m) + \\
& + min\big\{s_t^r + N_{t-1}^r, max\{s_t^r - I_{t-1}^r + D_{t-1}^r, 0\}\big\}. \\
.\Big(\Big(1 - \beta\big(min\{s_t^r + N_{t-1}^r, & max\{s_t^r - I_{t-1}^r + D_{t-1}^r, 0\}\}\big)\Big).p_t - c_t\Big) - \\
& - max\{min\{I_{t-1}^r - D_{t-1}^r, S_t^r\}, 0\}.\alpha.(p_{t-1} - p_t) + \\
& + \varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t), \qquad (5.4)
\end{aligned}
$$

where

$$A_t = \{p_t \in \mathbb{R}: p_t \le p_{t-1}, p_t \ge 0\}, \qquad (5.5)$$

$$\varphi_t(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t) = \mathbb{E}_{D_t}[Q_{t+1}(IL_t^m(I_{t-1}^m, D_{t-1}^m), IL_t^r(I_{t-1}^r, D_{t-1}^r), p_t, D_t)], \qquad (5.6)$$

$$IL_t^m(I_{t-1}^m, D_{t-1}^m) = max\{S_t^m, I_{t-1}^m - D_{t-1}^m\}, \qquad (5.7)$$

$$IL_t^r(I_{t-1}^r, D_{t-1}^r) = \begin{cases} s_t^r & if\ I_{t-1}^r - D_{t-1}^r \le s_t^r, \\ I_{t-1}^r - D_{t-1}^r & if\ s_t^r < I_{t-1}^r - D_{t-1}^r \le S_t^r, \\ S_t^r & if\ I_{t-1}^r - D_{t-1}^r > S_t^r. \end{cases} \qquad (5.8)$$

We already know the closed-form expression of the profit the manufacturer makes after the pricing and inventory decisions in the last period given the retailers' post-replenishments inventory levels, the retail price set by the manufacturer and the demand observed by the retailers in the last period. If we add a dummy period to the end of the selling horizon, we can call that expression the profit function of the dummy period. That profit function is as follows:

$$
\begin{aligned}
Q_{N+1}(I_N^m, I_N^r, p_N, D_N) = & - max\{I_N^m - D_N^m, 0\}.c_N.r + D_N^m.p_N - max\{D_N^m - I_N^m, 0\}.p_N + \\
& + max\{I_N^m - D_N^m, 0\}.w_{N+1}^m \\
& + max\{I_N^r - D_N^r, 0\}.(w_{N+1}^m - w_{N+1}^r). \qquad (5.9)
\end{aligned}
$$

The way how the variant SDDP algorithm is implemented for this price commitment policy is the same as the way how it is implemented for the price commitment policy discussed in Chapter 3.

5.3 Numerical Experiment

As we did for the previous price commitment contracts, we observe how the changes in some contractual parameters impact the approximately optimal price at which the manufacturer- controlled retailer should sell the product in the first period of the selling horizon of three periods and the retailers' true expected total profits. We employ the same discount rate function as in Section 4.3 and the same market share functions as in Section 2.4.

The contractual parameters examined in this section are minimum allowable discount rate, maximum allowable discount rate, discount factor, markdown rate, refund per product returned by the independent retailer at the end of the selling horizon, refund per product returned after the second period and reimbursement rate. Throughout the section, the values of some parameters are mostly fixed and unless otherwise stated, they are as shown in Table 5.2 provided below.

Table 5.2 The values of fixed contractual and non-contractual parameters

Parameter	Value
Holding cost per dollar per period ($)	0.05
Discount rate for backordered demand (%)	15
Salvage values in periods ($)	(60, 60, 60)
Production costs ($)	(60, 60, 60)
Manufacturer-controlled retailer's order-up-to levels	(22, 19, 17)
Independent retailer's order-up-to levels	(32, 20, 9)
Independent retailer's dispose-down-to levels	(32, 26, 12)
Mean number of potential customers in periods	(22, 19, 15)
Mean maximum-willingness-to-pay values for retailers ($)	(200, 175, 140)
Multinomial logit scale factors	(32.66, 27.45, 25.55)
Allowable amounts of backordered demand for retailers	(15, 15, 0)

We generate 15 Poisson random variates standing for the number of potential customers in the market for each period of the selling horizon and form 100 demand subscenarios in the forward step to obtain a lower bound for the optimal value of the actual SAA problem. By the stopping criterion we choose, the termination of the algorithm necessitates ten percent of the absolute value of the lower bound being larger than the difference between the upper bound and the lower bound.

The first contractual parameter we analyze in this section is the minimum allowable discount rate that the independent retailer asks for. For this purpose, we implement the SDDP algorithm by setting the discount factor to 1.03, the maximum discount rate to 40%, the markdown rate to 10%, the reimbursement rate to 70% and the refunds per product returned after the first, second and last period to 300$, 300$ and 100$, respectively. As presented in Table 5.3, the manufacturer is inclined to decrease the retail price as the minimum allowable discount rate increases to counteract the reduction in the revenue generated by selling the product to the independent retailer. Through such a pricing strategy, the manufacturer aims to attract some more potential customers that do not intend to purchase the product so as to raise the retailers' market shares. However, the retail price and the market shares do not change dramatically. That is, the manufacturer also avoids a sudden drop in the revenue arising from the reduction in the retail price.

Table 5.3 The change of the approximate optimal price and market shares in relation to the minimum allowable discount rate

Minimum allowable discount rate (%)	15	20	25	30	35
Approximate optimal price ($)	177.97	177.82	177.78	177.48	177.29
Proportion of lost customers (%)	11.36	11.32	11.31	11.23	11.18
Manufacturer's market share (%)	22.31	22.33	22.34	22.39	22.42
Independent retailer's market share (%)	66.33	66.35	66.35	66.38	66.40

As can be deduced from Figure 5.1, the confidence interval of the manufacturer's true expected total profit slightly shifts down and the confidence interval of the independent retailer's true expected total profit shifts up as the minimum allowable discount rate increases. The minimum allowable discount rate of 15% is the best option from the manufacturer's perspective but the minimum discount rate of 35% dominates all the other alternatives for the independent retailer. Therefore, the selection of the minimum allowable true expected total profits plays a significant role in determining a compromise value of the minimum allowable discount rate. The lower bound of the independent retailer's expected total profit is 2000$ and the lower bound of the manufacturer's expected total profit is 1500$ over the set of evaluated levels of the minimum allowable discount rate. If the manufacturer's profit expectation is above 2000$ or the independent retailer expects at least 2500$, then different values of the other contractual parameters should be negotiated.

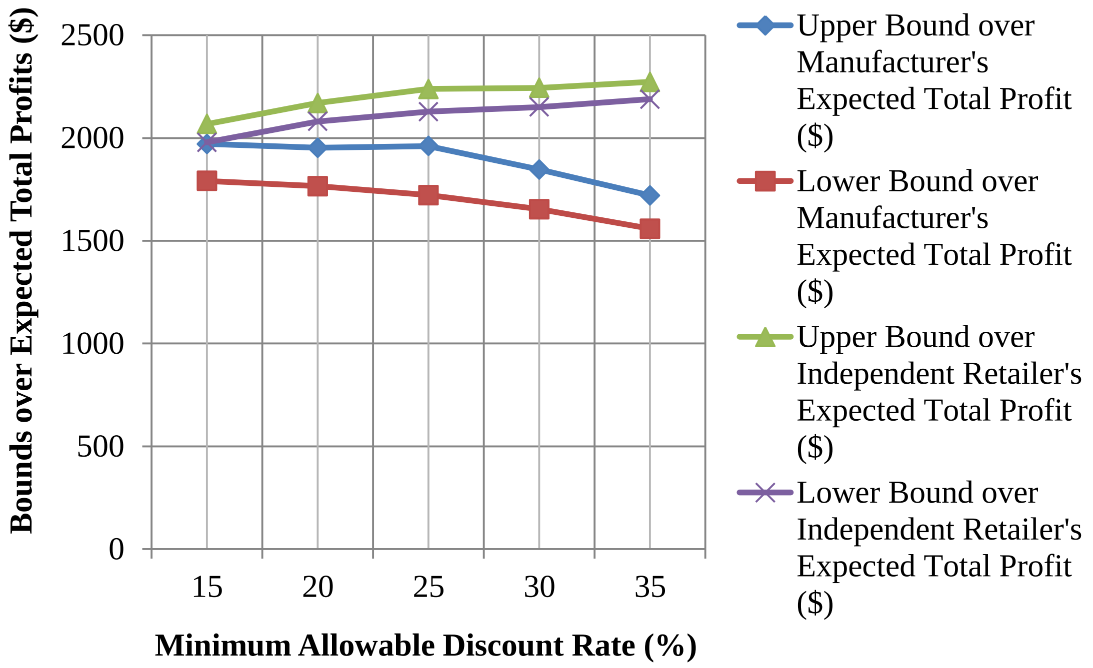

Figure 5.1 The evolution of the bounds over the retailers' expected total net profits in relation to the minimum allowable discount rate

The second parameter to be examined is the maximum allowable discount rate. For this purpose, we implement the SDDP algorithm by setting the discount factor to 1.03, the minimum discount rate to 25%, the markdown rate to 10%, the reimbursement rate to 70% and the refunds per product returned after the first, second and last period to 300$, 300$ and 100$, respectively. As in the case of the minimum allowable discount rate, the manufacturer tends to decrease the retail price as the maximum allowable discount rate increases as shown in Table 5.4 so as to raise the retailers' market shares and diminish the proportion of lost customers. Since the manufacturer mostly prefers the independent retailer attracting more customers to limit mid-life return, reimbursement and end-of-life return costs, the probability that the independent retailer observes a relatively high demand is not considerably low. For that reason, the change in the retail price is more dramatic compared to the change that occurs as the minimum allowable discount rate increases.

Table 5.4 The change of the approximate optimal price and market shares in relation to maximum allowable discount rate

Maximum allowable discount rate (%)	30	35	40	45	50
Approximate optimal price ($)	185.49	182.77	177.78	173.58	164.44
Proportion of lost customers (%)	13.49	12.68	11.31	10.26	8.26
Manufacturer’s market share (%)	21.03	21.49	22.34	23.04	24.55
Independent retailer’s market share (%)	65.48	65.83	66.35	66.70	67.19

As shown in Figure 5.2, an increase in the maximum allowable discount rate triggers a downward shift of the confidence interval of the manufacturer’s true expected total profit and the independent retailer’s expected total profit is negatively correlated with the manufacturer’s expected total profit. The maximum allowable discount rate of 30% is the most appealing option for the manufacturer but it is dominated by all the other alternatives from independent retailer’s perspective. Therefore, the independent retailer prefers a maximum allowable discount rate of 50% the most. If an expected total profit above 2000$ is sufficient for the independent retailer, then it is the best option to set the maximum allowable discount rate to 40%. If the manufacturer desires an expected total profit above 2000$, then the value of the maximum allowable discount rate should be less than 40%. If there is not any compromise value of the maximum allowable discount, then different values of the other contractual parameters should be negotiated or the retailers should revise their profit expectations.

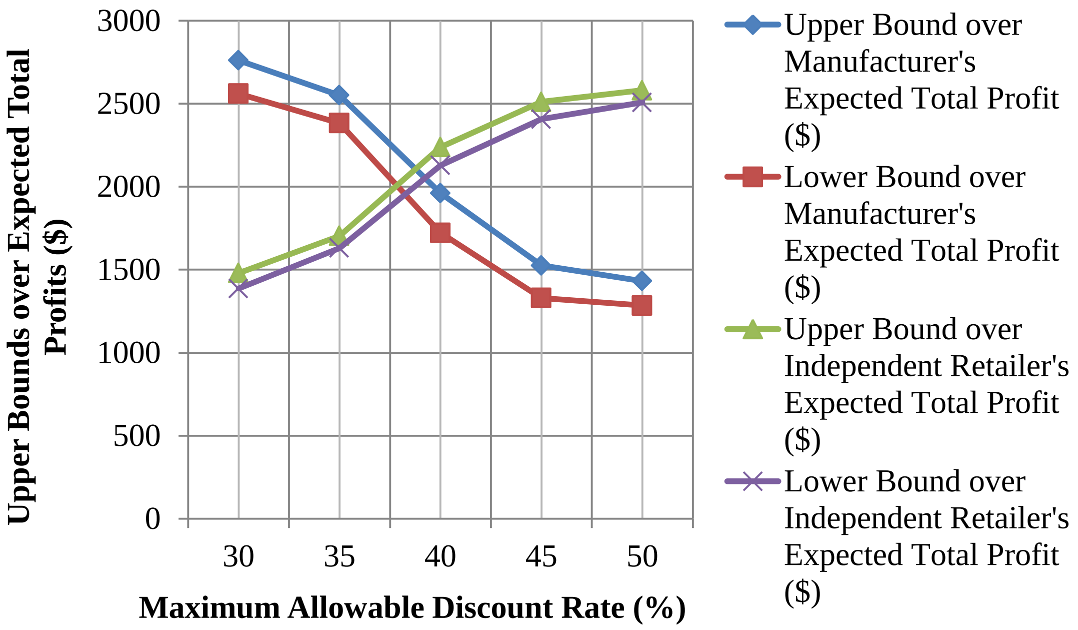

Figure 5.2 The evolution of the bounds over the retailers’ expected total net profits in relation to maximum allowable discount rate

The third contractual parameter that we examine is the discount factor. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 40%, the minimum allowable discount rate to 25%, the markdown rate to 20%, the reimbursement rate to 70% and the refunds per returned product after the first, second and third period to 300$, 100$ and 100$, respectively. As can be seen in Table 5.5, the manufacturer should lower the retail price to increase the retailers' market shares as the discount factor increases. In this way, the manufacturer tempts more potential customers into purchasing the product from any of the retailers so that a likely slump in the manufacturer's profit is at least partly counterbalanced. However, the change in the retail price and the retailers' market shares are not remarkably high.

Table 5.5 The change of the approximate optimal price and market shares in relation to discount factor

Discount factor	1.03	1.04	1.05	1.06	1.07
Approximate optimal price ($)	177.24	177.11	176.91	176.83	176.79
Proportion of lost customers (%)	11.17	11.14	11.09	11.07	11.06
Manufacturer's market share (%)	22.43	22.45	22.48	22.50	22.50
Independent retailer's market share (%)	66.40	66.41	66.43	66.43	66.44

As can be seen in Figure 5.3, the lower bound and the upper bound over the manufacturer's true expected total profit shows a decreasing trend as the discount factor increases. However, the graphs exhibiting the behaviors of the bounds over the independent retailer's true expected total profit are almost flat. We do not observe any sharp change in the retailers' profit values unlike the high sensitivity of the profit values to increasing discount factor in presence of the price commitment policy that does not contain mid-life returns but the special discount policy. Especially, the selection of the discount factor does not play a critical role in convincing the independent retailer to keep the inventory of the product. Therefore, it seems to be the best alternative to set the discount factor to 1.03 if a true expected total profit around 1500$ is satisfactory for the independent retailer. If there does not exist any compromise value of the discount factor that fulfills at least the retailers' minimum profit expectations, then the parties should bargain for different values of the other contractual parameters.

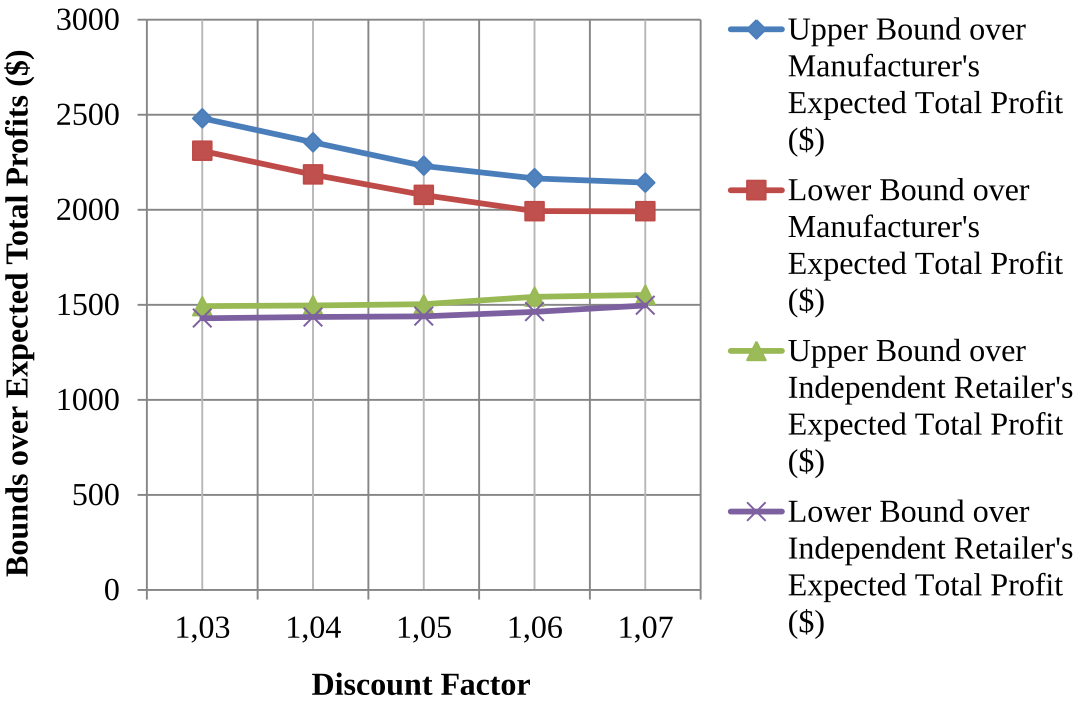

Figure 5.3 The evolution of the bounds over the retailers' expected total net profits in relation to discount factor

The fourth contractual parameter to be analyzed is the markdown rate. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 50%, the minimum allowable discount rate to 45%, the discount factor to 1.03, the reimbursement rate to 70% and the refunds per returned product after the first, second and third period to 300$, 100$ and 100$, respectively. As demonstrated in Table 5.6, the manufacturer shows a tendency to increase the retail price as the markdown rate rises. Since any increase in the markdown rate enables the independent retailer to attract more potential customers if the manufacturer would rather keep the price the same, the manufacturer prefers increasing the retail price to attain a higher earning per product sold to the end customer although this action causes its market share to drop to some extent.

Table 5.6 The change of the approximate optimal price and market shares in relation to markdown rate

Markdown rate (%)	0	10	20	30	40
Approximate optimal price ($)	156.99	160.64	163.33	168.99	188.57
Proportion of lost customers (%)	6.90	7.54	8.05	9.21	14.45
Manufacturer's market share (%)	25.75	25.16	24.73	23.80	20.50
Independent retailer's market share (%)	67.35	67.30	67.22	66.99	65.05

As can be inferred from Figure 5.4 shown below, the manufacturer's true expected total profit seems to increase as the markdown rate increases since the manufacturer's pricing strategy induces the manufacturer's earning per product sold to the end customer and the independent retailer to rise. On the contrary, this pricing strategy negatively impacts the independent retailer's profit. The markdown rate of 0% dominates all the other alternatives for the independent retailer but the markdown rate of 40% is the most alluring option for the manufacturer. For that reason, the retailers' minimum allowable expected profit values have a critical role in the selection of a compromise value of the markdown rate. For example, if a true expected total profit above 1000$ is adequate for the manufacturer, then the markdown rate can be set to a value around 20% to enable the independent retailer to achieve the highest possible expected total profit so that the independent retailer is lured into keeping the inventory of the product. However, if the manufacturer needs an expected total profit around 1500$, then it is the best option to set the markdown rate to 40% to ensure the profitability of the business for the manufacturer. However, the independent retailer's true expected total profit goes down below 1000$ in that case. This profit value might be undesirable for the independent retailer. If there is not any compromise value of the markdown rate, then different values of the other contractual parameters should be negotiated or the parties should revise their profit expectations.

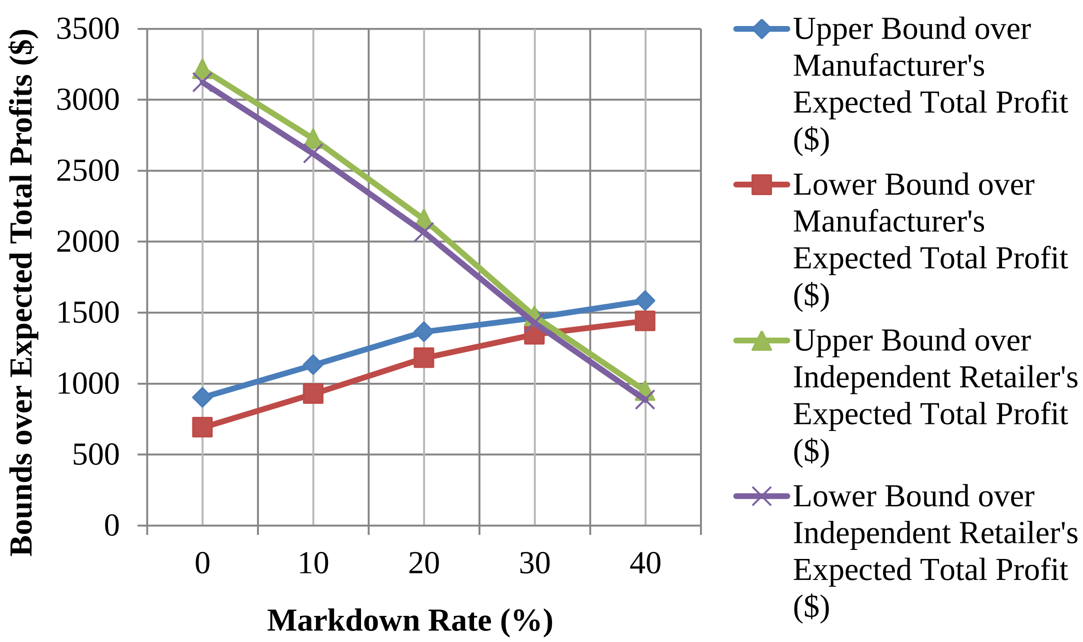

Figure 5.4 The evolution of the bounds over the retailers' expected total net profits in relation to markdown rate

The fifth contractual parameter that we observe is the refund per returned product after the last period of the selling horizon. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 40%, the minimum allowable discount rate to 25%, the discount factor to 1.03, the markdown rate to 20%, the reimbursement rate to 70% and the refund per returned product after each one of the first two periods to 300$, respectively. As can be seen in Table 5.7, the manufacturer should increase the retail price as the refund per product returned at the end of the selling horizon increases. We can state that any increase in the refund does not have a substantial effect on the independent retailer's market share as the market share slightly decreases. The manufacturer aims to compensate for any likely increase in the total cost stemming from the returns throughout the selling horizon by raising its earning per product sold to the end customer and the independent retailer. However, the manufacturer prefers increasing the retail price by a limited amount to avoid any massive adverse effect of the decrease in the retailers' market shares.

Table 5.7 The change of the approximate optimal price and market shares in relation to refund per returned product at the end of the selling horizon

Refund ($)	100	200	300	400	500
Approximate optimal price ($)	178.95	179.63	180.27	180.89	181.48
Proportion of lost customers (%)	11.62	11.80	11.98	12.15	12.32
Manufacturer's market share (%)	22.14	22.03	21.92	21.81	21.71
Independent retailer's market share (%)	66.24	66.17	66.10	66.04	65.97

The confidence interval of the manufacturer's true expected total profit moves down as the refund per returned product rises as exhibited in Figure 5.5 provided below. On the contrary, any increase in the refund leads to an upward shift of the confidence interval of the independent retailer's true expected total profit. A refund of 500$ is the most preferable alternative among all the discussed options for the independent retailer but it is the worst option for the manufacturer. A refund of 100$ dominates all the other options for the manufacturer. Just like for the previously analyzed contractual parameters, the retailers' minimum allowable true expected profits play a pivotal role in the determination of a compromise value of the refund per product. For example, if an expected total profit around 1500$ is acceptable for the manufacturer, then the refund per returned product can be set to 100$ to enable the independent retailer to make the highest expected total profit possible. If an expected total profit around 2000$ is satisfactory for the independent retailer, then the best option is to set the refund per product to 200$ to keep the business as profitable for the manufacturer as possible. If there is not any compromise value of the refund per product, then the parties should negotiate different values of the other contractual parameters or revise their profit expectations.

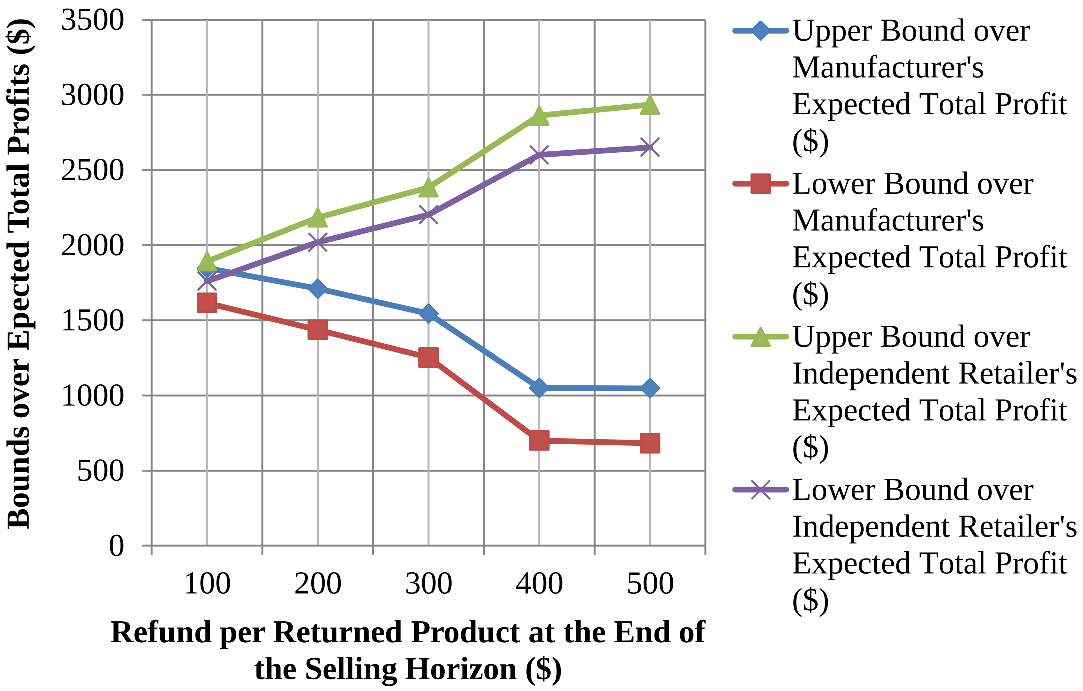

Figure 5.5 The evolution of the bounds over the retailers' expected total net profits in relation to refund per returned product at the end of the selling horizon

The sixth contractual parameter that we examine is the refund per product returned after the second period. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 40%, the minimum allowable discount rate to 25%, the discount factor to 1.03, the markdown rate to 20%, the reimbursement rate to 70% and the refunds per returned product after the first period and the last period to 300$ and 100$, respectively. The approximately optimal price increases as the refund increases as shown in Table 5.8 as is the case for the refund per returned product after the selling horizon.

Table 5.8 The change of the approximate optimal price and market shares in relation to refund per returned product after the second period

Refund ($)	100	200	300	400	500
Approximate optimal price ($)	177.24	178.12	178.95	179.76	180.54
Proportion of lost customers (%)	11.17	11.40	11.62	11.84	12.05
Manufacturer's market share (%)	22.43	22.28	22.14	22.00	21.87
Independent retailer's market share (%)	66.40	66.32	66.24	66.16	66.08

As can be inferred from Figure 5.6, the confidence interval of the manufacturer's true expected total profit shifts down as the refund increases and the independent retailer's true expected total profit is negatively correlated with the manufacturer's true expected total profit. The retailers should specify their minimum allowable expected total profits for the determination of a compromise value of the refund. For example, if an expected total profit of 1500$ is adequate for the manufacturer, then it is the best option to fix the refund at a value around 300$ to lure the independent retailer into being involved in the business. If the independent retailer desires an expected total profit around 2000$, then the refund can be set to a value around 400$. If there is no compromise value of the refund, then different values of the other contractual parameters should be negotiated.

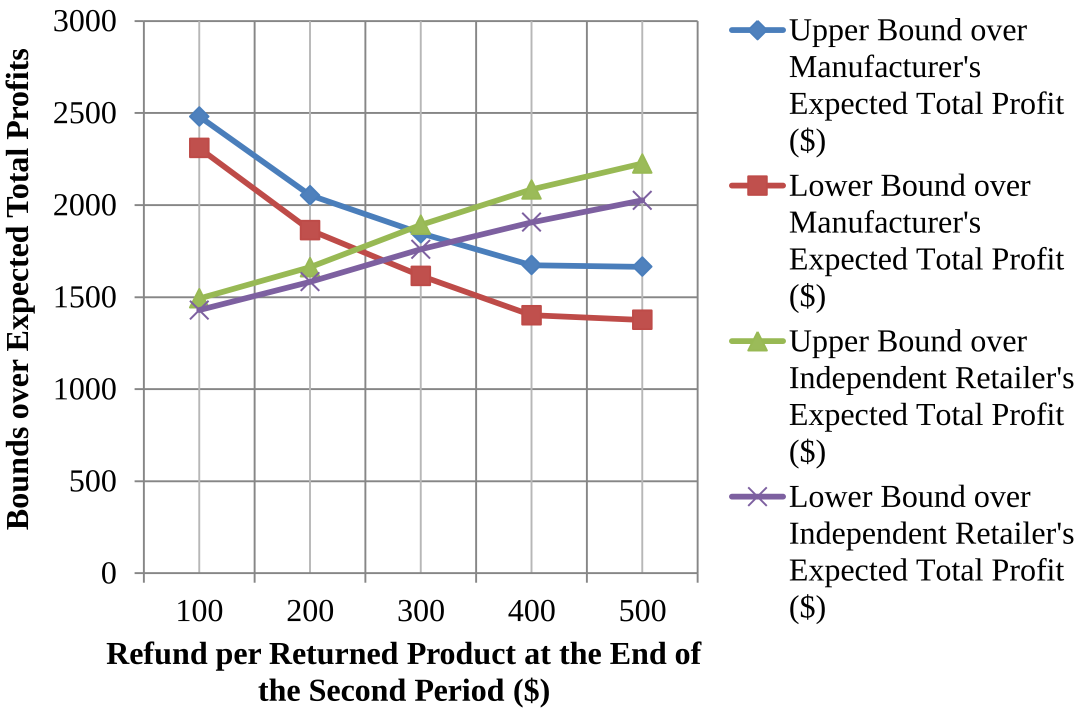

Figure 5.6 The evolution of the bounds over the retailers' expected total net profits in relation to refund per returned product at the end of the second period

The last contractual parameter that we examine is the reimbursement rate. For this purpose, we implement the SDDP algorithm by setting the maximum allowable discount rate to 40%, the minimum allowable discount rate to 25%, the discount factor to 1.03, the markdown rate to 20% and the refunds per returned product after the first, second and third period to 300$, 300$ and 100$, respectively. The manufacturer tends to reduce the retail price as the reimbursement rate increases as shown in Table 5.9. The manufacturer aims to increase the independent retailer's market share so that the independent retailer's expected on-hand stock after the first period lessens. Moreover, a decrease in the retail price in the first period also has an impact on the retail prices that will be set in the following periods because of the declining price environment. For that reason, the manufacturer is inclined to bring the retail prices closer to each other to refrain from high reimbursement costs. We can expect the difference between the retail prices in two consecutive periods to be smaller for higher values of the reimbursement rate.

Table 5.9 The change of the approximate optimal price and market shares in relation to reimbursement rate

Reimbursement rate (%)	20	40	60	80	100
Approximate optimal price ($)	202.97	194.83	183.43	170.17	156.96
Proportion of lost customers (%)	19.70	16.57	12.87	9.47	6.90
Manufacturer's market share (%)	17.98	19.41	21.38	23.61	25.76
Independent retailer's market share (%)	62.32	64.02	65.75	66.92	67.35

As can be deduced from Figure 5.7, the confidence interval of the manufacturer's true expected total profit shifts down as the reimbursement rate increases and the independent retailer's true expected total profit is negatively correlated with the manufacturer's true expected total profit. As explained in Section 4.3, the price protection does not have a substantial impact on the retailers' true expected total profits if the price commitment policy comprises price protection, end-of-life returns and special discount policy. For that reason, the selection of the reimbursement rate does not play a critical role in the profitability of the business for the retailers. However, if mid-life returns are also included in the price commitment policy, then the price protection is very effective in tempting the independent retailer into engaging in the business and the selection of a compromise value of the reimbursement rate is a critical task. For example, if an expected total profit above 2000$ is satisfactory for the manufacturer, then the reimbursement rate can be set to 80% to enable the independent retailer to make the highest possible profit. If an expected total profit above 1000$ is adequate for the independent retailer, then it is the best alternative to fix the reimbursement rate at approximately 40%. Even the values above 100% can be considered if the sides cannot compromise on any of these alternative values. If there does not exist any compromise value of the reimbursement rate, then different values of the other contractual parameters should be evaluated or the retailers should revise their profit expectations.

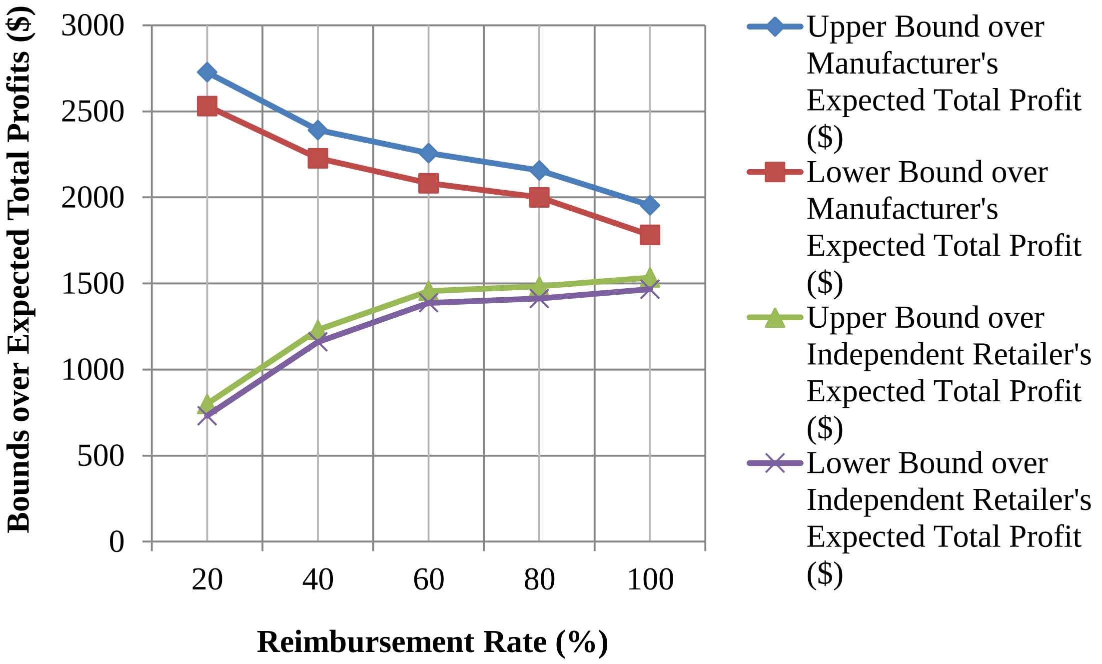

Figure 5.7 The evolution of the bounds over the retailers' expected total net profits in relation to reimbursement rate

The findings on the impact of an increase in the value of each critical contractual parameter on approximately optimal price and the retailers' true expected total net profits are summarized in Table 5.10 shown below.

In conclusion, the retailers' minimum allowable profit values have a critical role in the selection of the compromise values of each contractual parameter. However, minimum allowable discount rate and discount factor have a negligibly small impact on the retailers' expected total profits. Unlike the price commitment policy consisting of price protection, end-of-life returns and special discount policy, price protection is of a greater importance in presence of this policy including mid-life returns as a supplementary privilege to the independent retailer.

Table 5.10 Impact of an increase in the values of the critical contractual parameters on approximately optimal price and the retailers' true expected total net profits

	Approximately optimal price	Confidence interval of manufacturer's expected total net profit	Confidence interval of independent retailer's expected total net profit
Minimum allowable discount rate	Slightly decreasing	Shifting down	Shifting up
Maximum allowable discount rate	Decreasing	Shifting down	Shifting up
Discount factor	Slightly decreasing	First shifting down, then almost not shifting	Almost not shifting
Markdown rate	Increasing	Shifting up	Shifting down
Refund at the end of the selling horizon	Increasing	First shifting down, then almost not shifting	First shifting up, then almost not shifting
Refund at the end of the second period	Increasing	First shifting down, then almost not shifting	Shifting up
Reimbursement rate	Decreasing	Shifting down	First shifting up, then almost not shifting

Chapter 6

Conclusions and Future Prospects

6.1 Conclusions

If there is a price commitment policy consisting of price protection and end-of-life returns in effect, discount offered for the independent retailer's replenishment orders is a very effective privilege to protect the independent retailer financially. However, the changes in the retailers' expected total profits in relation to discount rate are very precipitous so the selection of an ideal value for the discount rate plays a pivotal role in ensuring the profitability for both parties. Markdown rate also has a massive impact on the retailers' expected total profits. The selections of values for markdown rate and discount rate should be carried out concurrently. Although a high refund per returned product and a high reimbursement rate do not seem quite substantial to lure the independent retailer into getting involved in the business, the existence of these two privileges in the price commitment contract can be regarded as being reassuring for the independent retailer. The selection of compromise values for reimbursement rate and refund per returned product is still significant to ensure high enough profitability for the retailers. The critical thing is to avoid inordinately high values of reimbursement rate since the independent retailer sustains financial loss in that case.

If the privileges provided for the independent retailer and the limitations of the business partnership are laid down by a price commitment contract containing mid-life returns as an additional privilege in conjunction with price protection and end-of-life returns, the retailers' true expected total profits are very sensitive to the refund per product returned at the end of the selling horizon. End-of-life returns are very effective in inducing the independent retailer to get involved in this business. However, mid-life returns do not seem as effective as end-of-life returns although it is still capable of increasing the profitability of the business for the independent retailer. The selection of ideal values for markdown rate, discount rate and reimbursement rate is very critical because the retailers' expected total profits are very sensitive to the values of these contractual parameters. The values for discount rate and markdown rate should be specified simultaneously. The approximately optimal price that the manufacturer should set in the first period of the selling horizon is also sensitive to the changes in reimbursement rate, discount rate and markdown rate.

If mid-life returns taking part in the second price commitment contract are substituted by special discount policy, then the effectiveness of price protection is disputable in terms of whether it renders the business more profitable for the independent retailer or not. The special discount policy is a favorable alternative to mid-life returns to convince the independent retailer into keeping the inventory of the product since the existence of the special discount policy raises the profitability for the independent retailer. Furthermore, the manufacturer has a chance to encourage the independent retailer to place replenishment orders of greater size in presence of special discount policy. This means that the special discount policy is beneficial for both of the parties. The selection of ideal values for discount rates, discount factor, markdown rate and refund per product returned at the end of the selling horizon is critical.

If the independent retailer is granted both special discount policy and mid-life return opportunities in conjunction with price protection and end-of-life return opportunity, minimum allowable discount rate and discount factor have a negligibly small impact on the retailers' expected total profits. Unlike the price commitment policy consisting of price protection, end-of-life returns and special discount policy, price protection is of a greater importance in presence of this policy including mid-life returns as a supplementary privilege to the independent retailer.

In the process of selecting ideal values for the contractual parameters, the retailers have to determine their minimum allowable profit values that they will stipulate in the contract negotiations because those values are very critical in the selection of compromise values of the contractual parameters. In the selection process, trade-offs between the retailers' expected total profits have to be reckoned with scrupulously, as well. Furthermore, the weights assigned to the retailers' expected total net profits have a conspicuous impact on compromise values. Therefore, whether the price protection contract is as profitable for both parties as expected or not depends on the retailers' profit expectations implying that the suitable selection of the contractual parameter values and the accuracy in the requirements of the price protection contract are very decisive. A comparison between the price commitment contracts discussed in this study in terms of the evaluation measures that we take into consideration is demonstrated in Appendix B.

The algorithm returns very reasonable results and considering the problem instances solved as the numerical experiments, the number of iterations required by the convergence of the algorithm is decent. For the solved problem instances, a single iteration of the algorithm lasts for approximately one and half an hour if it is implemented for the first problem and the third problem in which mid-life returns are not included. Otherwise, the algorithm takes around two and half an hour to converge. Of course, the values selected for the inventory replenishment policy parameters have an impact on the running time of a single iteration. However, since we study a case where a slow-moving *A* item is sold, we cannot expect large values of policy parameters. Furthermore, if the convergence of the algorithm entails more than one iteration, the running time of each additional iteration is shorter compared to the running times of the previous iterations. It is simply because of the upper bound constraints added to the constraint sets.

Compared to the approximate dynamic programming algorithms proposed to get around three curses of dimensionality, the running time per iteration is longer since the algorithm traverses all the possible pairs of inventory levels with which the retailers can start a given period instead of visiting a single state on each iteration. However, considering that approximate dynamic programming algorithms are bound to necessitate an undue number of iterations for a decent approximation because of the clumsy way how the value functions are updated from iteration to iteration, a longer running time per iteration of the variant SDDP algorithm is tolerable.

6.2 Societal Impact and Contribution to Global Sustainability

In high-tech industry, customers tend to purchase technologically advanced brand new products or the improved models of the products they already have. This tendency compels manufacturers to make some changes in their product mixes. With the development, production and introduction of some brand new products, the old products are offered at discounted prices to the customers that have relatively low budgets. As manufacturers can sell their products to the end customer via its own retailers, they may also prefer collaborating with some retailers in order to reach much more customers in the market. In this way, they both expand their markets and grant retailers an opportunity to make more profit. This implies that there is a partnership for the goals.

Manufacturers should offer some privileges to entice retailers into keeping the inventory of their products. Especially, external retailers want to be protected against sudden drops in the wholesale prices at which they purchase products. In this thesis, we study a selling environment where a manufacturer both runs its own retailer and collaborates with an independent retailer. The manufacturer offers the independent retailer price protection to keep its profit margins on decent levels. It also offers mid-life and end-of-life return opportunities to help the independent retailer avoid unduly high inventory holding costs and the disposal of leftover inventory. The concomitant mutual benefit contributes to decent work and economic growth.

We propose a modified version of the SDDP algorithm to propose an approximately optimal pricing strategy for the manufacturer as the manufacturer-controlled retailer and the independent retailer observe price-dependent stochastic demand throughout the selling horizon. By doing so, we also shed light on how the problems afflicted by three curses of dimensionality in which random event distribution depends on the decision variable can be dealt with in case parametric optimization is not possible. In this way, we contribute to industry, innovation and infrastructure.

6.3 Future Prospects

The retailers' expected total net profits can also be weighted differently and in that way, they can look for the best compromise solution. This approach also refers to multi-objective optimization. The objective functions of the models presented throughout the thesis can be reformed and the variant SDDP algorithm proposed in this study can be implemented to determine approximately Pareto optimal pricing strategy.

The assumptions can be relaxed to study some different cases. For example, we can relax the assumption on the inventory replenishment policies followed by the retailers throughout the selling horizon and allow them to determine their own replenishment policies. In this case, we have to optimize the amount of replenishment orders, as well. In this way, we can determine the optimal inventory replenishment policies and check whether or not the supply chain can be coordinated under each price commitment contract discussed in this study in a selling environment where retail and wholesale prices are not fixed. We can also allow the independent retailer to set the retail price at which it sells the product to the end customer by excluding RFM policy. In both cases, we have to determine Nash-equilibrium values so a different solution methodology has to be proposed.

We can also analyze a case where the manufacturer collaborates with more than two independent retailers and a case where a fast-moving *A* item or a *B* item is sold throughout the selling horizon. We can also study a problem in which a manufacturer collaborates with some retailers to sell multiple products. The existence of a lead time can also impact retailers' optimal inventory replenishment policies so we can also study a case where there exists a fixed or random lead time before the delivery of replenishment orders. In a much more advanced problem setting, a methodology can be proposed for continuous pricing and replenishment instead of periodic pricing and replenishment.

Furthermore, the variant SDDP algorithm can be improved in different aspects to reduce the number of iterations needed till convergence and the running time of each iteration. Some other existing methodologies can be evaluated in terms of whether they are applicable and adaptable to the problem studied in this paper and after the adaptation of another methodology, we can compare the variant SDDP algorithm to that new method in terms of efficiency.

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Appendix A

Step 0. Initialization:

Step 0a. Initialize the number of demand realizations M_j to be generated for all $j \in \{1,2,\dots,N\}$.

Step 0b. Generate a Poisson random variate $a_{j,m}$ with a mean of μ_j for all $m \in \{1,2,\dots,M_j\}$, $j \in \{1,2,\dots,N\}$.

Step 0c. Initialize the trial decision $\overline{p_j}$ for all $j \in \{1,2,\dots,N\}$ satisfying:

$$0 \le \overline{p_N} \le \overline{p_{N-1}} \le \cdots \le \overline{p_1}.$$

Step 0d. Set $LF^0_{j,l,n} = \{(p_j,T) \in \mathbb{R}^2 : p_j \ge 0\}$ for all $j \in \{1,2,\dots,N\}$, $l \in \{1,2,\dots,\max_{j\in\{1,2,\dots,N\}} S^m_j\}$, $n \in \{1,2,\dots,\max_{j\in\{1,2,\dots,N\}} S^r_j\}$.

Step 0e. Set $\bar{\vartheta}_N(l,n,p) = \frac{1}{M_N} . \sum_{x=1}^{M_N} \sum_{y=0}^{a_{N,x}} \sum_{z=0}^{a_{N,x}-y} Q_{N+1}(l,n,p,(y,z)).$

$$.\binom{a_{N,x}}{y}.\binom{a_{N,x}-y}{z}.(MS^m_N(p))^y.(MS^r_N(p))^z.$$

$$.(MS^o_N(p))^{a_{N,x}-y-z}.$$

Step 0f. Set k =1.

Step 1. Set $\bar{\vartheta}_t(l,n,p) = 0$ for all $t \in \{1,2,\dots,N-1\}$.

Step 2. Do for $s = S^m_{N-1}, S^m_{N-1}+1,\dots,\max_{j\in\{1,2,\dots,N-1\}} S^m_j$;

$t = S^r_{N-1}, S^r_{N-1}+1,\dots,\max_{j\in\{1,2,\dots,N-1\}} S^r_j$:

Step 2a. Do for $u = 1,2,\dots, M_{N-1}$; $v = 0,1,\dots, a_{N-1,u}$; $z = 0,1,\dots, a_{N-1,u} - v$:

Step 2aa. Set $mi = \max\{s-v, S^m_N\}$ and $ri = \max\{t-z, S^r_N\}$.

Step 2ab. Set

$$LF^k_{N,mi,ri} = LF^{k-1}_{N,mi,ri} \cap \{(p_N,T) \in \mathbb{R}^2 : T \le \bar{\vartheta}_N(mi,ri,p_N)\}.$$

Step 2ac. Solve the Lagrangian dual problem:

$$\min_{\lambda \ge 0} \max_{(p_N,T)\in LF^k_{N,mi,ri}} POD_N(t,p_N,z) + T + \lambda.(\bar{p}_{N-1} - p_N)$$

and let λ^* be the optimal solution of the Lagrangian dual problem

Step 2ad. Solve:

$$d^* = \max_{(p_N,T)\in LF^k_{N,mi,ri}} POD_N(t,p_N,z) + T - \lambda^*.p_N$$

Step 2ae. Update $\bar{\vartheta}_{N-1}(s,t,p)$ using:

$$\bar{\vartheta}_{N-1}(s,t,p) = \bar{\vartheta}_{N-1}(s,t,p) + \frac{1}{M_{N-1}}.\binom{a_{N-1,u}}{v}.\binom{a_{N-1,u}-v}{z}.$$

$$. (MS_{N-1}^m(p))^v . (MS_{N-1}^r(p))^z .$$
$$. (MS_{N-1}^o(p))^{a_{N-1,u}-v-z} .$$
$$. (PRD_N(s,t,p,v,z) + d^* + \lambda^* . p).$$

Step 2b. Set $u_{N-1}^k(s,t,p) = \bar{\vartheta}_{N-1}(s,t,p)$.

Step 3. Do for j = N-2, N-3,..., 1; $s = S_j^m$, $S_j^m + 1$,…,$\max_{g\in\{1,2,\dots,j\}} S_g^m$;

$t = S_j^r$, $S_j^r + 1$,…,$\max_{g\in\{1,2,\dots,j\}} S_g^r$:

Step 3a. Do for u = 1, 2,…, M_j; v = 0,1,…, $a_{j,u}$; z = 0,1,…, $a_{j,u}$ -v:

Step 3aa. Set $mi = \max\{s-v, S_{j+1}^m\}$ and $ri = \max\{t-z, S_{j+1}^r\}$.

Step 3ab. Set

$$LF_{j+1,mi,ri}^k = LF_{j+1,mi,ri}^{k-1} \cap$$
$$\cap \left\{(p_{j+1},T) \in \mathbb{R}^2 : T \leq u_{j+1}^k\left(mi, ri, p_{j+1}\right)\right\}.$$

Step 3ac. Solve the Lagrangian dual problem:

$$\min_{\lambda\geq 0} \max_{(p_{j+1},T)\epsilon LF_{j+1,mi,ri}^k} POD_{j+1}\left(t, p_{j+1}, z\right) + T + \lambda.\left(\bar{p}_j - p_{j+1}\right)$$

and let λ^* be the optimal solution of the Lagrangian dual problem.

Step 3ad. Solve:

$$d^* = \max_{(p_{j+1},T)\epsilon LF_{j+1,mi,ri}^k} POD_{j+1}\left(t, p_{j+1}, z\right) + T - \lambda^*. p_{j+1}.$$

Step 3ae. Update $\bar{\vartheta}_j(s,t,p)$ using:

$$\bar{\vartheta}_j(s,t,p) = \bar{\vartheta}_j(s,t,p) + \frac{1}{M_j}.\binom{a_{j,u}}{v}.\binom{a_{j,u}-v}{z}.\left(MS_j^m(p)\right)^v .$$
$$.\left(MS_j^r(p)\right)^z .\left(MS_j^o(p)\right)^{a_{j,u}-v-z} .$$
$$.\left(PRD_{j+1}(s,t,p,v,z) + d^* + \lambda^*.p\right).$$

Step 3b. Set $u_j^k(s,t,p) = \bar{\vartheta}_j(s,t,p)$.

Step 4. Determine a new candidate approximately optimal retail price for the first period:

Step 4a. Set $LF_{1,S_1^m,S_1^r}^k = LF_{1,S_1^m,S_1^r}^{k-1} \cap \{(p_1,T) \in \mathbb{R}^2 : T \leq u_1^k(S_1^m, S_1^r, p_1)\}$.

Step 4b. Solve:

$$\max_{(p_1,T) \in LF_{1,S_1^m,S_1^r}^k} FPP(p_1) + T$$

and let $\left(p_1^k, T^*\right)$ be the optimal solution and $\overline{\vartheta_k}$ be the optimal value of the problem.

Step 5. Uniformly choose K demand scenarios from the entire set of $\prod_{i=1}^{N} M_i$ scenarios with replacement and let D_j^w be the number of potential customers in the market in period j given wth demand scenario

Step 6. Set $\delta_w = FPP(p_1^k)$ for all $w \in \{1,2,\dots,K\}$.

Step 7. Do for w = 1,2,…,K:

Step 7a. Set $mi_1 = S_1^m$ and $ri_1 = S_1^r$.

Step 7b. Do for t = 1,2,…,N:

Step 7ba. Calculate the manufacturer-controlled retailer's and the independent retailer's market shares in the first period given the price p_t^k using the market share functions and let $MS_t^m(p_t^k)$ and $MS_t^r(p_t^k)$ be the manufacturer-controlled retailer's market share and the independent retailer's market share, respectively.

Step 7bb. Generate a binomial random variate od_m representing the demand observed by the manufacturer-controlled retailer given the probability of success of $MS_t^m(p_t^k)$ and the sample size of D_t^w.

Step 7bc. Generate a binomial random variate od_r representing the amount of demand observed by the independent retailer given the probability of success of $\frac{MS_t^r(p_t^k)}{1-MS_t^m(p_t^k)}$ and the sample size of $D_t^w - od_m$.

Step 7bd. If $t \neq N$, update the retailers' inventory levels after replenishments in the following period $t+1$:

$mi_{t+1} = \max\{mi_t\text{-}od_m, S_{t+1}^m\}$ and

$ri_{t+1} = \max\{ ri_t\text{-}od_r, S_{t+1}^r\}$.

Step 7be. If $t \neq N$, solve:

$$\max_{(p_{t+1},T) \in F_{t+1}} POD_{t+1}(ri_t, p_{t+1}, od_r) + \\ + PRD_{t+1}(mi_t, ri_t, p_t^k, od_m, od_r) + T$$

where

$F_{t+1} = LF_{t+1,mi_{t+1},ri_{t+1}}^k \cap \{(p_{t+1},T) \in \mathbb{R}^2 : p_{t+1} \leq p_t^k\}$

and let (p_{t+1}^k, T^*) be the optimal solution.

Step 7bf. If $t \neq N$, update the total profit by using:

$$\delta_w = \delta_w + POD_{t+1}\left(ri_t, p_{t+1}^k, od_r\right) +$$
$$+ PRD_{t+1}\left(mi_t, ri_t, p_t^k, od_m, od_r\right).$$

Step 7bg. If $t = N$, update the total profit by using:

$$\delta_w = \delta_w + Q_{N+1}\left(mi_N, ri_N, p_N^k, (od_m, od_r)\right).$$

Step 8. Construct a one-sided confidence interval with a confidence level of $1 - \alpha$ to determine a lower bound LB_k over the optimal value of the actual SAA problem:

$$LB_k = \sum_{w=1}^{K} \frac{1}{K} . \delta_w - z_\alpha . \frac{\sqrt{\left(\sum_{j=1}^{K}\left(\delta_j - \sum_{w=1}^{K}(1/K).\delta_w\right)^2\right)/K - 1}}{\sqrt{K}}.$$

Step 9. If $\overline{\vartheta_k} - LB_k < \epsilon$, then terminate the algorithm. p_1^k is the approximately optimal retail price for the first period.

Step 10. If $\overline{\vartheta_k} - LB_k \geq \epsilon$, then:

Step 10a. Set $\overline{p_j} = p_j^k$ for all $j \in \{1, 2, \ldots, N\}$.

Step 10b. Set $k = k + 1$ and return to Step 1.

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J1) B. Yildiz, M. Sutcu, A Variant SDDP Approach for Periodic-Review Approximately Optimal Pricing of a Slow-moving A Item in a Duopoly under Price Protection with End-of-life Return and Retail Fixed Markdown Policy published in the journal Expert Systems with Applications (Sep. 2022).

C1) B. Yildiz, M. Sutcu, Impact of Price Protection, Mid-life and End-of-life Returns on Pricing Strategies through a Modified SDDP Algorithm in Proc. of International Conference on Engineering and Applied Natural Sciences (Oct. 2022).